Amherst College
Department of Mathematics and Statistics

MATH 105 | MIDTERM 2 | FALL 2018

NAME: Solutions

Read This First!

- Keep cell phones off and out of sight.
- Do not talk during the exam.
- You are allowed one page of notes, front and back. No other books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Please read each question carefully. Show **ALL** work clearly in the space provided. There is an extra page at the back for additional scratchwork.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	6	Total
Points:	8	24	8	12	8	0	60
Score:							

1. [8 points] Consider the function

$$f(x) = 2x^3 + 5x.$$

Find an equation for the tangent line to the graph $y = f(x)$ at the point where $x = -1$.

$$f'(x) = 2\cdot 3x^2 + 5\cdot 1$$
$$= 6x^2 + 5$$
$$f'(-1) = 6\cdot(-1)^2 + 5$$
$$= 11.$$

We want the line through $(-1, f(-1)) = (-1, 2\cdot(-1)^3 + 5(-1))$
$= (-1, -7)$

w/ slope $f'(-1) = 11$.

ie.

$$(y+7) = 11(x+1)$$
$$= 11x + 11$$

$$\Leftrightarrow \boxed{y = 11x + 4}$$

2. [24 points] Evaluate the derivative of each function. **You do not need to simplify your answers.**

(a) $\dfrac{2x+1}{3x+1}$

$$f'(x) = \boxed{\frac{2\cdot(3x+1) - (2x+1)\cdot 3}{(3x+1)^2}}$$

(b) $\dfrac{2x^2+3\sqrt{x}+5}{\sqrt{x}} = 2x^{3/2} + 3 + 5x^{-1/2}$

$$f'(x) = 2\cdot\tfrac{3}{2}x^{1/2} + 0 + 5\cdot(-\tfrac{1}{2})x^{-3/2}$$

$$= \boxed{3x^{1/2} - \tfrac{5}{2}x^{-3/2}} \quad \left(\text{or } 3\sqrt{x} - \frac{5}{2x\sqrt{x}}\right)$$

(continued on reverse)

(c) $\sqrt{3+(1+x)^4}$

$$f'(x) = \frac{1}{2\sqrt{3+(1+x)^4}} \cdot \frac{d}{dx}\left(3+(1+x)^4\right)$$

$$= \boxed{\frac{1}{2\sqrt{3+(1+x)^4}} \cdot 4(1+x)^3} \cdot \cancel{\frac{d}{dx}(1+x)} \rightarrow 1$$

(d) $\dfrac{(2+3x)^2}{\sqrt{3-x}}$

$$f'(x) = \frac{2(2+3x)\cdot\frac{d}{dx}(2+3x)\cdot\sqrt{3-x} - (2+3x)^2\cdot\frac{1}{2\sqrt{3-x}}\cdot\frac{d}{dx}(3-x)}{\left(\sqrt{3-x}\right)^2}$$

$$= \boxed{\frac{2(2+3x)\cdot 3\cdot\sqrt{3-x} - (2+3x)^2\cdot\frac{1}{2\sqrt{3-x}}\cdot(-1)}{3-x}}$$

3. [8 points] Let

$$f(x) = \sqrt{1+x^2}.$$

Use the **limit definition** of the derivative to find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{1+(x+h)^2} - \sqrt{1+x^2}}{h} \cdot \frac{\sqrt{1+(x+h)^2} + \sqrt{1+x^2}}{\sqrt{1+(x+h)^2} + \sqrt{1+x^2}}$$

$$= \lim_{h\to 0} \frac{(1+(x+h)^2) - (1+x^2)}{h\cdot\left[\sqrt{1+(x+h)^2} + \sqrt{1+x^2}\right]}$$

$$= \lim_{h\to 0} \frac{\cancel{1} + \cancel{x^2} + 2xh + h^2 - \cancel{1} - \cancel{x^2}}{h\left[\sqrt{1+(x+h)^2} + \sqrt{1+x^2}\right]}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}\left[\sqrt{1+(x+h)^2} + \sqrt{1+x^2}\right]}$$

$$= \frac{2x+0}{\sqrt{1+(x+0)^2} + \sqrt{1+x^2}} = \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}$$

$$= \boxed{\frac{x}{\sqrt{1+x^2}}}$$

4. [12 points] Consider the function

$$f(x) = \frac{x^2+1}{2x+1}.$$

(a) Compute **and simplify** the derivative $f'(x)$.

$$f'(x) = \frac{2x(2x+1) - (x^2+1)\cdot 2}{(2x+1)^2}$$

$$= \frac{4x^2+2x-2x^2-2}{(2x+1)^2}$$

$$= \frac{2x^2+2x-2}{(2x+1)^2}$$

$$= \boxed{2\cdot\frac{x^2+x-1}{(2x+1)^2}}$$

(b) Compute **and simplify** the second derivative $f''(x)$. Your final answer should be $\frac{\cancel{18}\ 10}{(2x+1)^3}$. For full credit, show each step of your simplification.

$$f''(x) = 2\cdot\frac{(2x+1)(2x+1)^2 - (x^2+x-1)\cdot 2(2x+1)\cdot 2}{[(2x+1)^2]^2}$$

$$= 2\cancel{(2x+1)}\cdot\frac{(2x+1)^2 - 4(x^2+x-1)}{(2x+1)^{\cancel{4}\,3}}$$

$$= 2\cdot\frac{\cancel{4x^2}+\cancel{4x}+1-\cancel{4x^2}-\cancel{4x}+4}{(2x+1)^3}$$

$$= 2\cdot\frac{5}{(2x+1)^3} = \boxed{\frac{10}{(2x+1)^3}}$$

5. [8 points] At what points is the tangent line to the graph $y = (x+1)^2(2x-1)^3$ horizontal? For this problem, **it is enough to state the x-coordinate only in your answer.**

$$\frac{dy}{dx} = 2(x+1)\cdot 1\cdot (2x-1)^3 + (x+1)^2\cdot 3(2x-1)^2\cdot 2$$

$$= 2(x+1)(2x-1)^3 + 6(x+1)^2(2x-1)^2$$

$$= 2(x+1)(2x-1)^2\cdot\left[(2x-1) + 3(x+1)\right]$$

$$= 2(x+1)(2x-1)^2\cdot\left[5x+2\right]$$

so $\frac{dy}{dx} = 0 \iff \boxed{x = -1, \frac{1}{2}, \text{ or } -\frac{2}{5}}$

$$\begin{cases} x+1=0 & \iff x=-1 \\ 2x-1=0 & \iff 2x=1 \\ & \iff x = 1/2 \\ 5x+2=0 & \iff 5x=-2 \\ & \iff x=-2/5. \end{cases}$$

6. [3 points (bonus)] Evaluate and simplify

$$\frac{d}{dx}\sqrt{1+(5+\sqrt{x/6})^{12}}.$$

$$= \frac{1}{2\sqrt{1+(5+\sqrt{x/6})^{12}}}\cdot\frac{d}{dx}\left(1+(5+\sqrt{x/6})^{12}\right)$$

$$= \frac{1}{2\sqrt{1+(5+\sqrt{x/6})^{12}}}\cdot 12(5+\sqrt{x/6})^{11}\cdot\frac{d}{dx}(5+\sqrt{x/6})$$

$$= \frac{1}{2\sqrt{1+(5+\sqrt{x/6})^{12}}}\cdot 12(5+\sqrt{x/6})^{11}\cdot\frac{1}{2\sqrt{x/6}}\cdot\frac{d}{dx}(x/6)$$

$$= \frac{1}{2\sqrt{1+(5+\sqrt{x/6})^{12}}}\cdot\cancel{12}(5+\sqrt{x/6})^{11}\cdot\frac{1}{\cancel{2}\sqrt{x/6}}\cdot\frac{1}{\cancel{6}}$$

$$= \boxed{\frac{(5+\sqrt{x/6})^{11}}{2\sqrt{1+(5+\sqrt{x/6})^{12}}\cdot\sqrt{x/6}}}$$