

1. [9 points] Find all of the **critical numbers** of each function.

(a)  $f(x) = (x - 1)(x - 5)^3$

(b)  $f(x) = 10x^{4/5} - 5x^{9/5}$

2. [9 points] Evaluate each **limit at infinity**.

(a)  $\lim_{x \rightarrow \infty} \frac{3x^2 - 8x + 4}{6x^2 + 7x - 4}$

(b)  $\lim_{x \rightarrow -\infty} \frac{7x + 1}{2x^2 - 3x - 7}$

(c)  $\lim_{x \rightarrow \infty} \frac{x^2 + 7}{2x + 3}$

3. [10 points] Find the **absolute maximum and minimum** values of  $f(x) = x\sqrt{12 - x}$  on the interval  $[3, 11]$ .

4. [12 points] Sand is being poured onto a conical pile at a rate of 40 cubic feet per minute. The diameter of the pile is 4 times the height. How quickly (in feet per minute) is the radius of the pile increasing, when the radius is equal to 20 feet?

(The volume of a cone with height  $h$  and radius  $r$  is  $\frac{1}{3}\pi r^2 h$ .)

5. [20 points] Consider the function

$$f(x) = \frac{x^2 + 4}{x^2 + 12}.$$

The first two derivatives of this function are as follows. **You do not need to compute these.**

$$f'(x) = \frac{16x}{(x^2 + 12)^2}$$

$$f''(x) = -\frac{48(x^2 - 4)}{(12 + x^2)^3}$$

- (a) On which intervals is  $f(x)$  **increasing** on which intervals is it **decreasing**?
- (b) Using your answer to part (a), determine any **local max(s) and/or local min(s)** of  $f(x)$ . Give **both the  $x$  and  $y$  coordinates**.
- (c) On which intervals is  $f(x)$  **concave up** and on which intervals is it **concave down**?
- (d) Using your answer to part (c), determine any **point(s) of inflection** of  $f(x)$ . Give **both the  $x$  and the  $y$  coordinates**.
- (e) Determine any **horizontal asymptotes** of the function  $f(x) = \frac{x^2 + 4}{x^2 + 12}$ .
- (f) Draw a rough sketch of the graph  $y = f(x)$ , incorporating the information you found in parts (a) through (e).