Amherst College
Department of Mathematics and Statistics

MATH 105 2015 FINAL EXAM DECEMBER 18, 2015

NAME:_____

Read This First!

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Please read each question carefully. Show **ALL** work clearly in the space provided.
- If you need additional work space for a problem, please use the back of the **previous** page.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable. Pay careful attention to mathematical grammar.
- Answers must be clearly labeled in the spaces provided after each question.
- The exam consists of 10 questions which total to 200 points.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	30	30	15	10	20	20	20	20	15	20	200
Score:											

1. Evaluate the following limits. Be sure to show how you got your answer.

   (a) [6 points] $\lim_{x\to 2} \dfrac{x^2-5x+6}{x^2-4}$

   (b) [6 points] $\lim_{x\to 2} \dfrac{x-2}{\sqrt{2x}-2}$

   (c) [6 points] $\lim_{x\to+\infty} \dfrac{1+1000x}{10+x^2}$

   (d) [6 points] $\lim_{x\to 2^+} \dfrac{x}{x-2}$

   (e) [6 points] $\lim_{x\to 1} \dfrac{x^2-3x+2}{\dfrac{2}{x+3}-\dfrac{1}{x+1}}$

2. Compute the following derivatives. Do ***not*** simplify your answers.

   (a) [6 points] $\dfrac{d}{dx}\left(x^2+x\sqrt{x}+2\sqrt{2}\right)$.

   (b) [6 points] Suppose $g(t)$ is a function satisfying $g(1)=16$ and $g'(1)=8$. Then define $f(t)$ by the formula

   $$f(t)=t^2\sqrt{g(t)}.$$

   Compute $f'(1)$.

   (c) [6 points] $\left(\pi(x^2+x)^3(7x-1)^{3/2}\right)'$.

   (d) [6 points] Compute $\dfrac{d^2}{dx^2}\left(x^3+\dfrac{1}{x^3}\right)$

   (e) [6 points] Suppose $y=\left(x+\sqrt{x^2+x^3}\right)^4$. Compute $\dfrac{dy}{dx}$.

3. Let $f(x)=\sqrt{x^2+1}$.

   (a) [10 points] Compute $f'(x)$ using the limit definition of derivative.

   (b) [5 points] Compute $f'(x)$ using the usual rules of differentiation.

4. The position of an object moving on the number line is $s(t)=t^3-\frac{3}{2}t^2-21t+1$ ft, where t is measured in seconds.

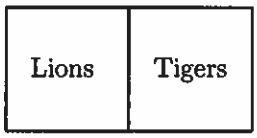
   (a) [5 points] Compute the velocity and acceleration of the moving object. You do *not* need to simplify your answers.

   (b) [5 points] Find all times (positive and negative) at which the velocity is zero.

5. [20 points] Draw the graph of the curve $y=x^3-12x$. Please indicate where the function is increasing and decreasing and where the graph is concave up and concave down. Also label local and absolute maxs and mins and points of inflection, and pay attention to intercepts and symmetry.

6. [20 points] A person inflates a spherical balloon by pumping in air at a rate of 60 in^3/min. How fast is the radius of the balloon increasing when the balloon is 10 inches across? The volume of a sphere of radius r is $V=\frac{4}{3}\pi r^3$.

7. [20 points] A small zoo has 1200 ft of fence to create an enclosure for their lions and tigers. Since the two types of animals don't get along, the fence includes a divider down the middle:



Find the dimensions of the enclosure that give the animals as much space as possible. Give a careful solution and be sure to explain how you found the interval.

8. To compensate for the lack of snow, let's pretend that there is a two hour snowstorm whose intensity varies during the course of the storm. Suppose that the total amount of snow as of time t is given by

$$S(t) = 3t^2 - t^3 \text{ inches of snow,}$$

where t is times (in hours) since the storm began.

(a) [5 points] Find the average rate of at which snow was falling between $t = 0$ and $t = 2$ hours.

(b) [5 points] How fast was snow falling exactly 30 minutes after the storm began?

(c) [10 points] Find the absolute maximum of the rate of snowfall during the time interval $0 \leq t \leq 2$ hours.

9. Let $f(x) = \dfrac{x^3}{(x-1)^2}$.

(a) [5 points] Compute $f'(x)$ and simplify your answer.

(b) [4 points] Find the three critical numbers of $f(x)$.

(c) [2 points] For one of the critical numbers found in part (b), the question "local minimum, local maximum, or neither" is not relevant. Tell me which critical number this is and why the question is not relevant.

(d) [4 points] For the other critical numbers found in part (b), determine whether they are a local minimum, local maximum, or neither.

10. Consider the function

$$g(x) = \begin{cases} 2 - x^2 & x \leq 0 \\ 1 + \dfrac{1}{x} & x > 0. \end{cases}$$

(a) [8 points] Draw the graph of $g(x)$.

(b) [8 points] Use the graph drawn in part (a) to find the following limits:

$\lim_{x \to 0^+} g(x) =$

$\lim_{x \to 0^-} g(x) =$

$\lim_{x \to 0} g(x) =$

$\lim_{x \to +\infty} g(x) =$

(c) [4 points] On the graph you drew in part (a), indicate where it is concave up and concave down. Does the concavity change anywhere? If the answer is yes, does this mean that the graph has a point of inflection? Explain.