1 (a) $$\lim_{x\to 2}\frac{x^2-5x+6}{x^2-4} = \lim_{x\to 2}\frac{\cancel{(x-2)}(x-3)}{(x+2)\cancel{(x-2)}} \overset{DSP}{=} \frac{2-3}{2+2} = \boxed{-1/4}$$

(b) $$\lim_{x\to 2}\frac{x-2}{\sqrt{2x}-2} = \lim_{x\to 2}\frac{x-2}{\sqrt{2x}-2}\cdot\frac{\sqrt{2x}+2}{\sqrt{2x}+2}$$

$$= \lim_{x\to 2}\frac{(x-2)(\sqrt{2x}+2)}{2x-4} = \lim_{x\to 2}\frac{\cancel{(x-2)}(\sqrt{2x}+2)}{2\cancel{(x-2)}}$$

$$= \frac{\sqrt{2\cdot 2}+2}{2} = \frac{4}{2} = \boxed{2}$$

(c) $$\lim_{x\to\infty}\frac{1+1000x}{10+x^2} = \lim_{x\to\infty}\frac{1+1000x}{10+x^2}\cdot\frac{1/x^2}{1/x^2}$$

$$= \lim_{x\to\infty}\frac{1/x^2+1000/x}{10/x^2+1} = \frac{1/\infty+1000/\infty}{10/\infty+1}$$

$$= \frac{0}{1} = \boxed{0}$$

(d) $$\lim_{x\to 2^+}\frac{x}{x-2} = \frac{2}{0^+} = \boxed{\infty}$$

(e) $$\lim_{x\to 1}\frac{x^2-3x+2}{\frac{2}{x+3}-\frac{1}{x+1}} = \lim_{x\to 1}\frac{x^2-3x+2}{\frac{2}{x+3}\cdot\frac{x+1}{x+1}-\frac{1}{x+1}\cdot\frac{x+3}{x+3}}$$

$$= \frac{(x-1)(x-2)}{\frac{(2x+2)-(x+3)}{(x+1)(x+3)}} = \frac{\cancel{(x-1)}(x-2)(x+1)(x+3)}{\cancel{x-1}}$$

$$\overset{DSP}{=} \frac{(1-2)(1+1)(1+3)}{1} = -1\cdot 2\cdot 4 = \boxed{-8}$$

② (a) $\frac{d}{dx}\left(x^2+x\sqrt{x}+2\sqrt{2}\right) = \frac{d}{dx}x^2+\frac{d}{dx}x^{3/2}+\frac{d}{dx}2\sqrt{2}$

$= 2x+\frac{3}{2}x^{1/2}+0$

$= \boxed{2x+\frac{3}{2}\sqrt{x}}$

(b) $g(1)=16 \quad g'(1)=8$

$f(t)=t^2\sqrt{g(t)}$

$f'(t)=2t\sqrt{g(t)}+t^2\cdot\frac{1}{2\sqrt{g(t)}}g'(t)$

$\Rightarrow f'(1)=2\cdot 1\cdot\sqrt{16}+1^2\cdot\frac{1}{2\sqrt{16}}\cdot 8 = 2\cdot 4+\frac{1}{8}\cdot 8$

$= 8+1 = \boxed{9}$

(c) $\left(\pi\left(x^3+x\right)^3\left(7x-1\right)^{3/2}\right)'$

$= \pi\cdot\left[\left(x^3+x\right)^3\right]'\left(7x-1\right)^{3/2}+\pi\left(x^3+x\right)^3\left[\left(7x-1\right)^{3/2}\right]'$

$= \boxed{\pi\cdot 3\left(x^3+x\right)^2\cdot\left(3x^2+1\right)\cdot\left(7x-1\right)^{3/2}+\pi\left(x^3+x\right)^3\cdot\frac{3}{2}\left(7x-1\right)^{1/2}\cdot 7}$

(d) $\frac{d^2}{dx^2}\left(x^3+\frac{1}{x^3}\right)=\frac{d}{dx}\left(3x^2-3\cdot\frac{1}{x^4}\right)$

$= \boxed{6x+12\cdot\frac{1}{x^5}}$

(e) $\frac{d}{dx}\left(x+\sqrt{x^2+x^3}\right)^4 = 4\left(x+\sqrt{x^2+x^3}\right)^3\cdot\frac{d}{dx}\left(x+\sqrt{x^2+x^3}\right)$

$= \boxed{4\left(x+\sqrt{x^2+x^3}\right)^3\cdot\left(1+\frac{1}{2\sqrt{x^2+x^3}}\cdot\left(2x+3x^2\right)\right)}$

② (a) $f(x)=\sqrt{x^2+1}$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}\cdot\frac{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}$$

$$=\lim_{h\to 0}\frac{(x+h)^2+\cancel{1}-x^2-\cancel{1}}{h\cdot\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}\overset{DSP}{=}\frac{2x+0}{\sqrt{(x+0)^2+1}+\sqrt{x^2+1}}=\frac{2x}{2\sqrt{x^2+1}}$$

$$=\boxed{\frac{x}{\sqrt{x^2+1}}}$$

(b) $$\frac{d}{dx}\sqrt{x^2+1}=\frac{1}{2\sqrt{x^2+1}}\cdot\frac{d}{dx}(x^2+1)=\frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x$$

$$=\frac{x}{\sqrt{x^2+1}}\ \checkmark$$

4 (a)

$$s(t) = t^3 - \frac{3}{2}t^2 - 21t + 1$$

$$v(t) = s'(t) = 3t^2 - 3t - 21$$

$$a(t) = v'(t) = 6t - 3$$

(b) $v(t) = 0 \iff 3(t^2 - t - 7) = 0$

$$\iff t = \frac{1 \pm \sqrt{(-1)^2 - 4\cdot(-7)}}{2} = \boxed{\frac{1 \pm \sqrt{29}}{2}}$$

⑤ $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12 = 3(x^2-4) = 3(x+2)(x-2)$

crit. numbers -2 & $+2$.

	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
$3(x+2)$	$-$	$+$	$+$
$(x-2)$	$-$	$-$	$+$
f'	$+$	$-$	$+$
f	↗	↘	↗

local max @ $(-2, f(-2)) = (-2, 16)$

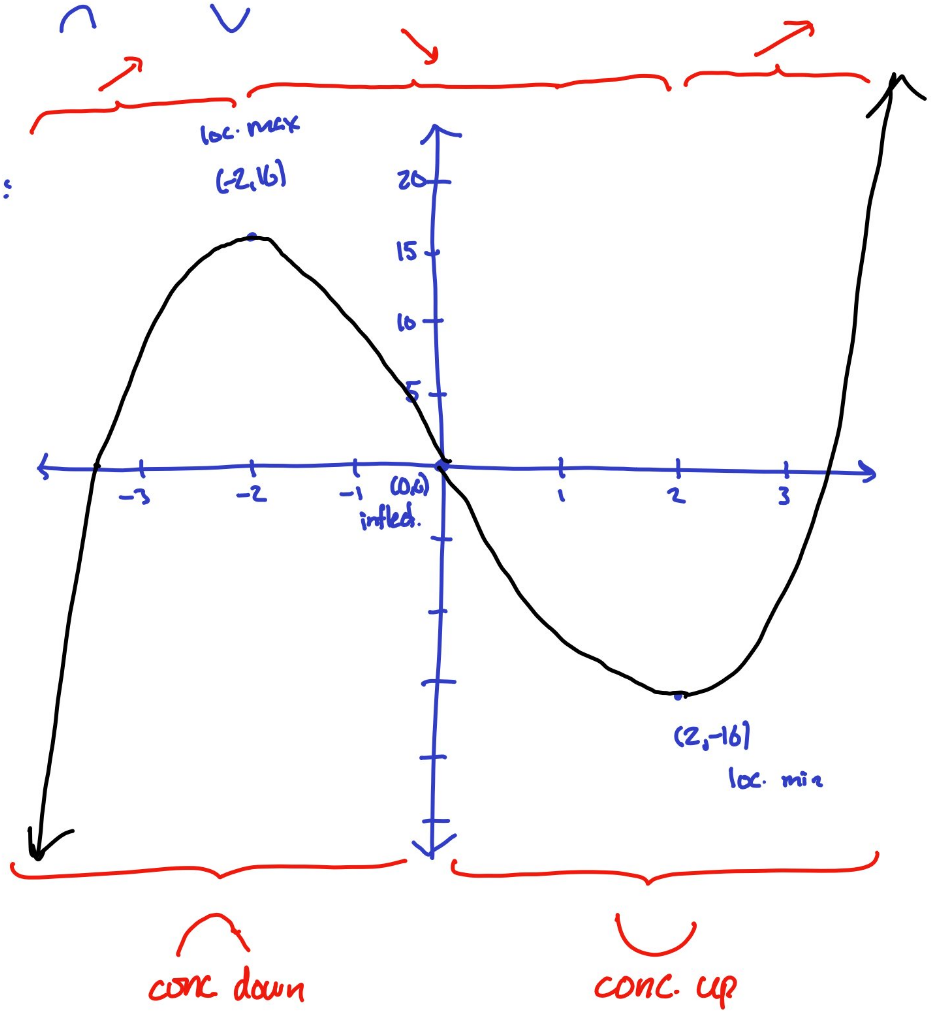
local min @ $(2, f(2)) = (2, -16)$

$f''(x) = 6x$

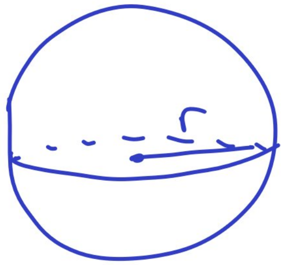
	$(-\infty,0)$	$(0,\infty)$
f''	$-$	$+$
f	$\cap$	$\cup$

inflection pt. @ $(0, f(0)) = (0,0)$.

Sketch:



6



$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow V' = \frac{4}{3}\pi \cdot 3r^2 \cdot r'$$

At key moment:

$$60 = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} \cdot 5^2 \cdot r'$$

$$= 100\pi r'$$

$$\Rightarrow r' = \frac{60}{100\pi} = \boxed{\frac{3}{5\pi}}$$

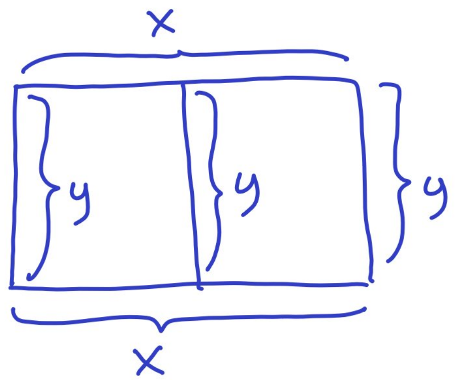
radius is growing by $\frac{3}{5\pi}$ in. per minute.

At key moment:

$V' = 60$ (in^3/min)

$r = 5$ (in.) since diam$=10$

7



amount of fence $= 2x + 3y = 1200$

Area $= A = xy$

We can solve for y to write A as a function of x alone:

$$3y = 1200 - 2x$$
$$y = 400 - \tfrac{2}{3}x$$

$$A(x) = x \cdot \left(400 - \tfrac{2}{3}x\right)$$

feasible values of x: $\quad x \geq 0$ & $y \geq 0$

$\iff x \geq 0$ & $\tfrac{2}{3}x \leq 400$

$\iff x \geq 0$ & $x \leq 600$

So the interval is $[0, 600]$.

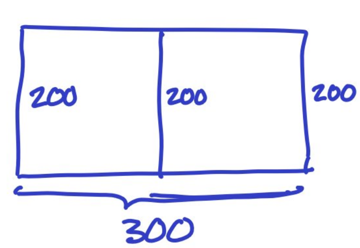
We want the max of $A(x) = x\left(400 - \tfrac{2}{3}x\right)$ on $[0, 600]$. We can use the closed interval method:

$$A'(x) = 1 \cdot \left(400 - \tfrac{2}{3}x\right) + x \cdot \left(-\tfrac{2}{3}\right) = 400 - \tfrac{4}{3}x$$

$\Rightarrow$ crit. pt. where $\quad 400 = \tfrac{4}{3}x \iff \underline{x = 300}$

$$A(0) = 0 \cdot 400 = 0$$
$$A(300) = 300 \cdot 200 = 60{,}000 \leftarrow \underline{\text{max.}}$$
$$A(600) = 600 \cdot 0 = 0$$

So the optimal dimensions are $x = 300$, $y = 200$



8 $S(t) = 3t^2 - t^3$

(a) avg. rate $= \dfrac{\text{total snowfall}}{\text{time}} = \dfrac{S(2) - S(0)}{2-0}$

$$= \frac{3\cdot 2^2 - 2^3 - 0}{2} = \frac{4}{2} = \boxed{2} \text{ inches/hour.}$$

(b) $S'(t) = 6t - 3t^2$

rate @ 30 min. = rate @ $t = \frac{1}{2}$ = $6\cdot\frac{1}{2} - 3\cdot\left(\frac{1}{2}\right)^2 = 3 - 3/4$

$= \boxed{\frac{9}{4}}$ in./hr.

(c) We want the maximum value of $S'(t) = 6t - 3t^2$ on $[0,2]$.
Using the closed interval method:

$S''(t) = 6 - 6t$, which is 0 @ $t=1$.

Test $t=1$ & endpoints:

$S'(0) = 6\cdot 0 - 3\cdot 0 = 0$

$S'(1) = 6\cdot 1 - 3\cdot 1 = 3$ ← max.

$S'(2) = 6\cdot 2 - 3\cdot 4 = 0$

So the snow was falling fastest at $\underline{\underline{t=1}}$ (halfway through the storm), when it was falling at an instantaneous rate of 3 inches per hour.

(9) $f(x) = \dfrac{x^3}{(x-1)^2}$

(a) $f'(x) = \dfrac{3x^2(x-1)^2 - x^3 \cdot 2(x-1)}{((x-1)^2)^2} = \dfrac{x^2(x-1)\cdot[3(x-1)-2x]}{(x-1)^4}$

$= \dfrac{x^2\cancel{(x-1)}(x-3)}{(x-1)^{\cancel{4}3}} = \dfrac{x^2(x-3)}{(x-1)^3}$

(b) num. is 0 @ $x=0$ & $x=3$ | Critical numbers are

undef. (denom 0) @ $x=1$ | $\underline{0, 1, \& 3}$.

(c) The "local max/min/neither" question is not relevant at $\underline{x=1}$, since $f(x)$ itself is not defined there.

(d) Sign analysis.

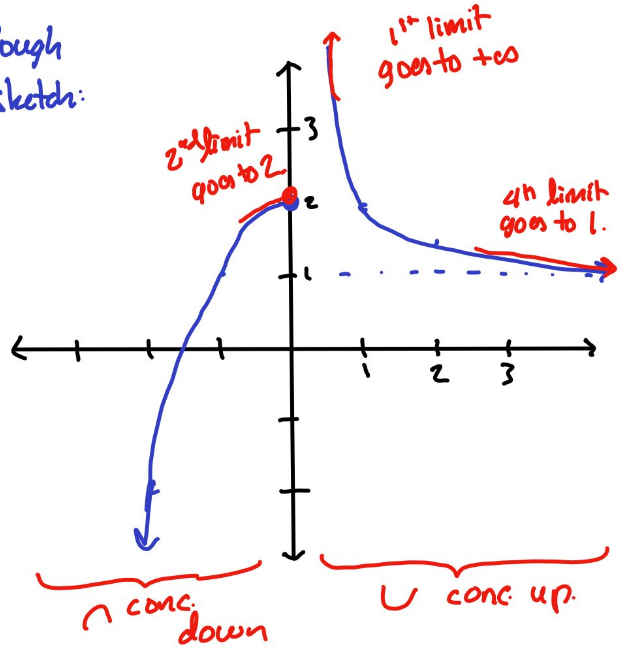
	0	1	3	
x^2	+	+	+	+
$(x-3)$	−	−	−	+
$\frac{1}{(x-1)^3}$	−	−	+	+
f'	+	+	−	+
f	↗	↗	↘	↗

1st deriv. test:

$\boxed{\begin{array}{l} x=0 \text{ is neither a max nor a min. } (\nearrow\nearrow) \\ x=3 \text{ is a local min. } (\searrow\nearrow) \end{array}}$

(10) $g(x) = \begin{cases} 2-x^2 & x \le 0 \\ 1 + \frac{1}{x} & x > 0 \end{cases}$

(a) Rough sketch:



How to draw this: $\underline{2}-x^2$ is like $y=x^2$, but vertically flipped & then translated up 2 units. (draw for $x \le 0$)

$1+\frac{1}{x}$ is like $y=\frac{1}{x}$, but translated up one unit.

(b) $\lim_{x \to 0^+} g(x) = +\infty$ (see red marks above for how to see these in the graph).

$\lim_{x \to 0^-} g(x) = 2$

$\lim_{x \to 0} g(x)$ does not exist, since the one-sided limits disagree.

$\lim_{x \to \infty} g(x) = \underline{1}$.

(c) As drawn above, the graph is concave down for $x<0$ & concave up for $x>0$.

The concavity changes @ $x=0$, but this is <u>not</u> considered an inflection point since it is a discontinuity of the graph (infinite discontinuity).