Матн	105			,	TEST	27		
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Name:_____

Read This First!

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted. Cell phones out of sight.

FALL 2015

- Please read each question carefully. Show ALL work clearly in the space provided. You may
 use the backs of pages for additional work space.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable
- Answers must be clearly labeled in the spaces provided after each question.
- The exam consists of Questions 1-8, which total to 100 points.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	6	7	8	Total
Points:	15	14	10	15	16	10	15	5	100
Score:									

- 1. [15 points] Compute the following derivatives.
 - (a) $\frac{d}{dx} \left(\frac{x^2 + \pi^2}{x^3 + \sqrt{7}^3} \right)$. Do not simplify your answer.

(b) Let $f(u) = \frac{h(u)}{u^2 + 1}$, where h(2) = -1 and h'(2) = 3. Compute f'(2).

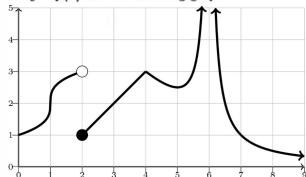
(c) $((x^2+1)^3(1-3x)^2)'$. Do not simplify your answer.

- 2. [14 points]
 - (a) State the limit definition of the derivative of a function f(x).

(b) Compute $\frac{d}{dx} \left(\frac{1}{x^2 + 1} \right)$ using the limit definition of derivative.

3. [10 points] Compute the second derivative of $f(x) = \frac{x^2}{x+3}$ and simplify your answer.

4. [15 points] Suppose that y = f(x) has the following graph:



(a) For which numbers a does f(x) fail to be continuous at a? Give reasons using the definition of continuity?

(b) For which numbers a does f(x) fail to be differentiable at a? Give reasons.

(c) Find all x's for which f'(x) > 0.

5. [16 points] We are adding trash to a brand new landfill. Assume that the amount of trash in the landfill at time t (= months since the landfill opened) is given by the formula

$$W(t) = 100t + 10t^2$$
 tons of trash.

(a) How much trash was added to the landfill during the first six months of its operation?

(b) Compute the rate of adding trash during this six month time period.

(c) What was the rate of adding trash exactly six months after the landfill opened?

(d) When you compare the answers to parts (b) and (c), what conclusion do you draw?

6. [10 points] Find the equation of the line tangent to the curve $y = \frac{x^2 + \sqrt{x} + 1}{2 - x}$ at the point where the x-coordinate is equal to 1.

7. [15 points] Let

$$f(x) = \frac{x^2}{\sqrt{x^2 - 1}}.$$

Note that f(x) is defined when $x^2 > 1$, which holds when either x > 1 or x < -1.

(a) Compute f'(x) and simplify your answer as much as possible. Your final answer should be $f'(x) = \frac{x(x^2 - 2)}{(x^2 - 1)^{3/2}}$. To get full credit, I need to see every step of the simplification.

(b) Find all points on the curve where the tangent line is horizontal.

8. [5 points] The production q of a company depends on both the capital investment K (in dollars) and the size of the labor force L (the number of workers). In economics, one frequently used formula for q in terms of K and L is the Cobb-Douglas production function

$$q = \sqrt{KL}$$
.

Assuming the capital investment remains constant, compute the rate of change of production as the number of workers increases.

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