Amherst College

DEPARTMENT OF MATHEMATICS

Math 105

Midterm Exam #3

December 6, 2013

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify your answers if required.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		15
2		20
3		20
4		15
5		20
6		10
Total		100

1. [15 Points] **Critical Numbers**

(a) Find critical numbers for the function $f(x) = \dfrac{x^2 + 1}{x - 3}$.

(b) Find the critical numbers for $f(x) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$.

2. [20 Points] **Absolute Extreme Values**

(a) Find the absolute maximum and absolute minimum values of

$$G(x) = (x-3)^2(x+2)^3 \quad \text{on} \quad [0,4].$$

(b) Find the absolute maximum and absolute minimum values of

$$F(x) = x\sqrt{4-x^2} \quad \text{on} \quad [-1,2].$$

3. [20 Points]   **Related Rates**

A conical paper cup of water is 4 inches across the entire top and 5 inches deep. It has a hole in the bottom point and is leaking water at 2 cubic inches per second. At what rate is the height of the water level decreasing when the water height is 1 inch?

$$*** \text{ Recall the volume of the cone is given by } V = \frac{1}{3}\pi r^2 h ***$$

4. [15 Points]   **Limits at Infinity**

(a) $\displaystyle\lim_{x \to \infty} \frac{x^9 + 8x^7 + 6x^5 + 4}{3x^2 + 1}$

(b) $\displaystyle\lim_{x \to -\infty} \frac{1 - x^3}{7x^3 + x^2 - 100}$

(c) $\displaystyle\lim_{x \to \infty} \frac{x^2 - x + 1}{2x^5 + 7x^2 + 3}$

5. [20 Points]   **Curve Sketching**   Let $f(x) = \dfrac{-x^2 + x + 2}{x^2 - 2x + 1}$.

For this function, discuss domain, vertical and horizontal asymptotes, intervals of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

Take my word for it that (you do **NOT** have to compute these)

$$f'(x) = \frac{x - 5}{(x - 1)^3} \quad \text{and} \quad f''(x) = \frac{-2x + 14}{(x - 1)^4}.$$