

Name: Solutions

- Keep phones off and out sight.
- No calculators, notes, books, or other aids.
- Do not talk during the quiz.
- Show all work.

Differentiation rules reference (this will not be printed on the midterm; include them on your note sheet!):

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$(f/g)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

1. Differentiate each function using the differentiation rules.

(a) $f(x) = 7x^3 - 2x + 3$

$$f'(x) = 7 \cdot 3x^2 - 2 + 0$$

$$= \boxed{21x^2 - 2}$$

(b) $f(x) = \frac{x+2}{x^2+3}$

$$f'(x) = \frac{(x+2)'(x^2+3) - (x+2)(x^2+3)'}{(x^2+3)^2} = \frac{1 \cdot (x^2+3) - (x+2) \cdot 2x}{(x^2+3)^2}$$

$$= \frac{x^2+3-2x^2-4x}{(x^2+3)^2} = \boxed{\frac{-x^2-4x+3}{(x^2+3)^2}}$$

(c) $f(x) = \sqrt{5x-2}$

$$f'(x) = \frac{1}{2\sqrt{5x-2}} \cdot \frac{d}{dx}(5x-2) = \boxed{\frac{5}{2\sqrt{5x-2}}}$$

2. Find all points on the graph $y = x^3 - 3x$ where the tangent line is horizontal. Each point should be specified as a pair of coordinates (x, y) .

$$\frac{dy}{dx} = 3x^2 - 3$$

t. line is horizontal when $3x^2 - 3 = 0$

$$\Leftrightarrow 3(x^2 - 1) = 0$$

$$\Leftrightarrow 3(x+1)(x-1) = 0$$

$$\Leftrightarrow x = \pm 1.$$

So the points are $(1, 1^3 - 3 \cdot 1)$ & $(-1, (-1)^3 - 3(-1))$,
ie. $(1, -2)$ & $(-1, 2)$.

3. Find an equation for the tangent line to the curve $y = x\sqrt{x}$ at the point where $x = 4$.

$$y = x^{3/2}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

slope at $x=4$ is $\frac{3}{2} \cdot 4^{1/2} = 3$.

& y -coord. is $4\sqrt{4} = 8$.

Tangent line:

$$(y-8) = 3(x-4)$$

or $y-8 = 3x-12$, ie. $y = 3x-4$