WORKSHEET 3: SOLUTIONS

1. $$\lim_{x\to 2} \frac{x^2-4}{x-2} = \frac{0}{0} \quad \text{So} \quad \lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2} x+2 = \boxed{4}$$

$$\lim_{x\to 1} \frac{x^2-4}{x-2} = \frac{1-4}{1-2} = \frac{-3}{-1} = \boxed{3}$$

The second one was easier to find b/c we didn't need to factor.

2. $$\lim_{x\to 2^-} \frac{x^2+6x+8}{x-2} = \frac{4+12+8}{0^-} = \frac{24}{0^-} \rightarrow \boxed{-\infty}$$

3. $$\lim_{x\to 2} \frac{x^2+5x-14}{x^2-4x+12} = \frac{4+10-14}{4-8+12} = \frac{0}{8} = \boxed{0}$$

4. $$\lim_{x\to 2} \frac{x^2+5x-14}{x^2-8x+12} \overset{0/0}{=} \lim_{x\to 2} \frac{(x+7)\cancel{(x-2)}}{(x-6)\cancel{(x-2)}} = \lim_{x\to 2} \frac{x+7}{x-6} = \frac{2+7}{2-6} = \boxed{\frac{9}{-4}}$$

5. $$\lim_{x\to 0} \frac{x+1}{x(x+2)} \longrightarrow \lim_{x\to 0^+} \frac{x+1}{x(x+2)} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to 0^-} \frac{x+1}{x(x+2)} = \frac{1}{0^-} = -\infty$$

Since these are different, $\lim_{x\to 0} \frac{x+1}{x(x+2)}$ DNE

6. $$\lim_{x\to(-3)^+} \frac{x^2+4x+3}{x^2-2x-15} = \frac{9-12+3}{9+6-15} = \frac{0}{0}, \quad \text{So, } \lim_{x\to(-3)^+} \frac{x^2+4x+3}{x^2-2x-15} = \lim_{x\to(-3)^+} \frac{\cancel{(x+3)}(x+1)}{(x-5)\cancel{(x+3)}}$$

$$= \lim_{x\to(-3)^+} \frac{x+1}{x-5} = \frac{-3+1}{-3-5} = \frac{-2}{-8} = \boxed{\frac{1}{4}}$$

7. $$\lim_{x\to -3} \frac{x^2+4x+3}{x^2+6x+9} = \frac{9-12+3}{9-18+9} = \frac{0}{0}, \quad \text{So } \lim_{x\to -3} \frac{x^2+4x+3}{x^2+6x+9} = \lim_{x\to -3} \frac{\cancel{(x+3)}(x+1)}{(x+3)\cancel{(x+3)}}$$

$$= \lim_{x\to -3} \frac{(x+1)}{(x+3)} = \frac{-2}{0} \Rightarrow \lim_{x\to -3^-} \frac{(x+1)}{(x+3)} = \frac{-2}{0^-} = +\infty$$

$$\lim_{x\to -3^+} \frac{x+1}{x+3} = \frac{-2}{0^+} = -\infty$$

Since these are diff. $\lim_{x\to -3} \frac{x^2+4x+3}{x^2+6x+9}$ $\boxed{\text{DNE}}$

8. $$\lim_{t\to 1} \frac{t-1}{g(t^2)-3}, \quad g(t) = 2t+1 \longrightarrow \lim_{t\to 1} \frac{t-1}{g(t^2)-3} = \lim_{t\to 1} \frac{t-1}{(2t^2+1)-3} = \lim_{t\to 1} \frac{t-1}{2t^2-2}$$

$$= \lim_{t\to 1} \frac{t-1}{2(t^2-1)} = \lim_{t\to 1} \frac{\cancel{t-1}}{2(t+1)\cancel{(t-1)}} = \lim_{t\to 1} \frac{1}{2(t+1)} = \boxed{\frac{1}{4}}$$

$$= \frac{\frac{1}{9} - \frac{1}{9}}{0} = \frac{0}{0}$$

9. $$\lim_{x\to -5} \frac{\frac{1}{4-x} - \frac{1}{9}}{x+5} = \lim_{x\to -5} \frac{\frac{9}{9(4-x)} - \frac{4-x}{9(4-x)}}{x+5} = \lim_{x\to -5} \frac{\frac{9-4+x}{9(4-x)}}{x+5}$$

$$= \lim_{x\to -5} \frac{\cancel{5+x}}{9(4-x)} \cdot \frac{1}{\cancel{x+5}} = \lim_{x\to -5} \frac{1}{9(4-x)} = \frac{1}{9(9)} = \boxed{\frac{1}{81}}$$

10. $$\lim_{x\to -3} \frac{x^2-4x-21}{\sqrt{1-x}-2} = \frac{9+12-21}{\sqrt{1+3}-2} = \frac{0}{0}$$

$$\hookrightarrow \lim_{x\to -3} \frac{x^2-4x-21}{\sqrt{1-x}-2} \cdot \frac{\sqrt{1-x}+2}{\sqrt{1-x}+2} = \lim_{x\to -3} \frac{(x^2-4x-21)(\sqrt{1-x}+2)}{(\sqrt{1-x})^2-4}$$

$$= \lim_{x\to -3} \frac{(x^2-4x-21)(\sqrt{1-x}+2)}{-3-x} \overset{0/0}{=} \lim_{x\to -3} \frac{(x-7)\cancel{(x+3)}(\sqrt{1-x}+2)}{-\cancel{(x+3)}}$$

$$= \lim_{x\to -3} \frac{(x-7)(\sqrt{1-x}+2)}{-1} = \frac{(-3-7)(\sqrt{4}+2)}{-1} = -(-10)(4) = \boxed{40}$$

11. $$f(x) = \frac{1}{x} \Rightarrow \lim_{t\to 2} \frac{f(t-1)-2f(t)}{t^2-4} = \lim_{t\to 2} \frac{\frac{1}{t-1} - 2\left(\frac{1}{t}\right)}{t^2-4}$$

$$= \lim_{t\to 2} \frac{\frac{t}{t(t-1)} - \frac{2(t-1)}{t(t-1)}}{t^2-4} = \lim_{t\to 2} \frac{\frac{t-2t+2}{t(t-1)}}{t^2-4} = \lim_{t\to 2} \frac{-t+2}{t(t-1)} \cdot \frac{1}{t^2-4}$$

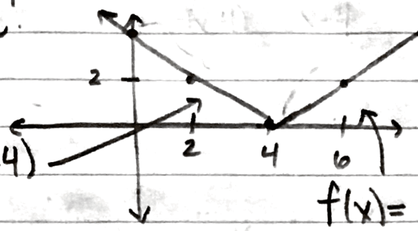
$$= \lim_{t\to 2} \frac{-\cancel{(t-2)}}{t(t-1)} \cdot \frac{1}{(t+2)\cancel{(t-2)}} = \lim_{t\to 2} \frac{-1}{t(t-1)(t+2)} = \frac{-1}{2(1)(4)} = \boxed{\frac{-1}{8}}$$

12. $$g(x) = \sqrt{x} \Rightarrow \lim_{s\to 1} \frac{g(s^2+8)-3}{s-1} = \lim_{s\to 1} \frac{\sqrt{s^2+8}-3}{s-1} = \frac{\sqrt{9}-3}{0} = \frac{0}{0}$$

$$= \lim_{s\to 1} \frac{\sqrt{s^2+8}-3}{s-1} \cdot \frac{\sqrt{s^2+8}+3}{\sqrt{s^2+8}+3} = \lim_{s\to 1} \frac{s^2+8-9}{(s-1)(\sqrt{s^2+8}+3)} = \lim_{s\to 1} \frac{\cancel{(s-1)}(s+1)}{\cancel{(s-1)}(\sqrt{s^2+8}+3)}$$

$$= \lim_{s\to 1} \frac{s+1}{\sqrt{s^2+8}+3} = \frac{2}{\sqrt{9}+3} = \frac{2}{6} = \boxed{\frac{1}{3}}$$

13. $\lim\limits_{x\to 4^-} \dfrac{|x-4|}{x-4}$ } Note: For this problem, think of what $|x-4|$ looks like.



$f(x) = -(x-4)$ $f(x) = x-4$

What is it doing as $x \to 4^-$? What is it doing as $x \to 4^+$?

So, $\lim\limits_{x\to 4^-} \dfrac{|x-4|}{x-4} = \lim\limits_{x\to 4^-} \dfrac{-\cancel{(x-4)}}{\cancel{x-4}}$ $\boxed{= -1}$

14. $\lim\limits_{x\to 4^+} \dfrac{|x-4|}{x-4} = \lim\limits_{x\to 4^+} \dfrac{x-4}{x-4} = \boxed{1}$

15. $\lim\limits_{x\to 3} \dfrac{\frac{x}{x-2} - \frac{x+6}{x}}{x-3} = \dfrac{\frac{3}{1} - \frac{9}{3}}{0} = \dfrac{0}{0}$

$= \lim\limits_{x\to 3} \dfrac{\frac{x(x)}{x(x-2)} - \frac{(x+6)(x-2)}{x(x-2)}}{x-3} = \lim\limits_{x\to 3} \dfrac{\frac{x^2 - (x^2+4x-12)}{x(x-2)}}{x-3} = \lim\limits_{x\to 3} \dfrac{\frac{-4x+12}{x(x-2)}}{x-3}$

$= \lim\limits_{x\to 3} \dfrac{-4\cancel{(x-3)}}{x(x-2)} \cdot \dfrac{1}{\cancel{x-3}} = \lim\limits_{x\to 3} \dfrac{-4}{x(x-2)} = \dfrac{-4}{3(3-2)} = \boxed{\dfrac{-4}{3}}$

16. $\lim\limits_{x\to 3} \dfrac{-1}{(x-3)^2} = \dfrac{-1}{0} \Rightarrow$ So, $\lim\limits_{x\to 3^-} \dfrac{-1}{(x-3)^2} = \dfrac{-1}{(0^-)^2} = -\infty$

$\lim\limits_{x\to 3^+} \dfrac{-1}{(x-3)^2} = \dfrac{-1}{(0^+)^2} = -\infty$

} Since they're the same, $\lim\limits_{x\to 3} \dfrac{-1}{(x-3)^2}$ $\boxed{-\infty}$