WORKSHEET 4 - SOLUTIONS:

Recall: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

1. (a) $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{5-6(1+h)+4(1+h)^2-(5-6+4)}{h}$

$= \lim_{h\to 0} \frac{\cancel{5}-\cancel{6}-6h+4(1+2h+h^2)\cancel{-5}\cancel{+6}-4}{h} = \lim_{h\to 0} \frac{-6h\cancel{+4}+8h+4h^2\cancel{-4}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2+4h)}{\cancel{h}} = \lim_{h\to 0} 2+4h = \boxed{2}$

(b) By part (a), slope is $\underline{2}$.
$x = \underline{1} \Rightarrow f(1) = 5-6+4 = \underline{3}$

$y-3 = 2(x-1)$
$y = 2x-2+3$
$\boxed{y = 2x+1}$

2(a) $f'(x) = \lim_{h\to 0} \frac{(x+h)^3-x^3}{h}$

$= \lim_{h\to 0} \frac{(x+h)(x^2+2xh+h^2)-x^3}{h} = \lim_{h\to 0} \frac{\cancel{x^3}+2x^2h+xh^2+x^2h+2xh^2+h^3\cancel{-x^3}}{h}$

$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h} = \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}} = \lim_{h\to 0} 3x^2+3xh+h^2 = \boxed{3x^2}$

(b) $f'(x) = \lim_{h\to 0} \frac{\sqrt{(x+h)}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim_{h\to 0} \frac{\cancel{x}+h\cancel{-x}}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$

(c) $f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{-1}{x^2}}$

(d) $f'(x) = \lim_{h\to 0} \dfrac{\dfrac{x+h+1}{x+h-1} - \dfrac{x+1}{x-1}}{h} = \lim_{h\to 0} \dfrac{\dfrac{(x-1)(x+h+1)-(x+h-1)(x+1)}{(x+h-1)(x-1)}}{h}$

$= \lim_{h\to 0} \dfrac{(\cancel{x^2}+\cancel{xh}+\cancel{x}-\cancel{x}-h-\cancel{1})-(\cancel{x^2}+\cancel{x}+\cancel{xh}+h-\cancel{x}-\cancel{1})}{(x+h-1)(x-1)} \cdot \dfrac{1}{h}$

$= \lim_{h\to 0} \dfrac{-2\cancel{h}}{(x+h-1)(x-1)\cancel{h}} = \lim_{h\to 0} \dfrac{-2}{(x+h-1)(x-1)} = \boxed{\dfrac{-2}{(x-1)^2}}$

(e) $f'(x) = \lim_{h\to 0} \dfrac{\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \dfrac{\dfrac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} \cdot \dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} = \lim_{h\to 0} \dfrac{\dfrac{x-(x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}}{h}$

$= \lim_{h\to 0} \dfrac{-\cancel{h}}{\cancel{h}(\sqrt{x}\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})} = \lim_{h\to 0} \dfrac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \dfrac{-1}{(\sqrt{x})^2(\sqrt{x}+\sqrt{x})} = \boxed{\dfrac{-1}{2x\sqrt{x}}}$

[Note! $2x\sqrt{x} = 2\sqrt{x^3}$ because $2\sqrt{x\,x^2}$ would cause x^2 to come out as x]

So, $\dfrac{-1}{2x\sqrt{x}} = \dfrac{-1}{2\sqrt{x^3}}$.

3(a) $9x^2y + 2xy^3 = \boxed{xy(9x+2y^2)}$

(b) $2(x+1)^2y^3 - 8(x+1)y^5 = \boxed{2(x+1)y^3\left[(x+1)-4y^2\right]}$

(c) $3(x+1)^2(1-2x)^4 + (x+1)^3 4(1-2x)^3(-2)$

$= (x+1)^2(1-2x)^3\left[3(1-2x) + (x+1)4(-2)\right] = (x+1)^2(1-2x)^3\left[3-6x-8x-8\right]$

$= \boxed{(x+1)^2(1-2x)^3(-5-14x)}$

4(a) REMEMBER $f'(x)$ is the slope of the tangent line at x.

So $f'(x)=0$ when the tangent line slope is FLAT.

$f'(x)>0$ " " is positive

$f'(x)<0$ " " is negative.

Therefore, $f'(x)=0$ when $x=2, 6$;

$f'(x)>0$ for x in $[0,2)$ and $(6,\infty)$. $f'(x)<0$ for x in $(2,6)$.

(b) See graph for sketches.

Approximations will vary!!

$f'(0) = 2$

$f'(2) = 0$

$f'(4) = -2$

$f'(6) = 0$

$f'(8) = 0.90$

$f'(10) = 0.95$

$f'(x)$ for $x > 6$, should be increasing SLIGHTLY as x gets larger.

(c) See graph for rough sketch!