WORKSHEET 5 SOLUTIONS

1) $$f'(x) = \frac{(x+1)'(x-1) - (x+1)(x-1)'}{(x-1)^2} = \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2}$$

$$= \boxed{\frac{-2}{(x-1)^2}}$$

2) $$y'(x) = \frac{-2}{(x-1)^2}\ ;\quad \text{NOTE: } y(3) = \frac{3+1}{3-1} = \frac{4}{2} = \boxed{2} \quad \text{AND} \quad y'(3) = \frac{-2}{(3-1)^2} = \boxed{\frac{-1}{2}}$$

So, we have $(3,2)$ and $y'(3) = -1/2$, which gives us $y-2 = -1/2(x-3) \Rightarrow y-2 = -1/2\,x + 3/2$

$$\boxed{y = -1/2\,x + 7/2}$$

3) $$f'(x) = (x^2+7)'\sqrt{4x+1} + (x^2+7)(\sqrt{4x+1})' \quad \leftarrow \text{this was given!}$$

$$= 2x\sqrt{4x+1} + (x^2+7)\,2/\sqrt{4x+1}$$

$$= \frac{\sqrt{4x+1}\,(2x)\sqrt{4x+1}}{\sqrt{4x+1}} + \frac{2(x^2+7)}{\sqrt{4x+1}} = \frac{2x(4x+1)+2x^2+14}{\sqrt{4x+1}}$$

$$= \frac{8x^2+2x+2x^2+14}{\sqrt{4x+1}} = \frac{10x^2+2x+14}{\sqrt{4x+1}}$$

4) $$\frac{d}{dx}\left(\frac{2x+1}{3x+1}\right) = \frac{(2x+1)'(3x+1) - (2x+1)(3x+1)'}{(3x+1)^2} = \frac{2(3x+1)-3(2x+1)}{(3x+1)^2}$$

$$= \frac{\cancel{6x}+2-\cancel{6x}-3}{(3x+1)^2} = \boxed{\frac{-1}{(3x+1)^2}}$$

5) $$f(x) = \frac{3x^4}{x\sqrt{x}} + \frac{2x^2}{x\sqrt{x}} - \frac{7x}{x\sqrt{x}} + \frac{1}{x\sqrt{x}} = \frac{3x^4}{x^{3/2}} + \frac{2x^2}{x^{3/2}} - \frac{7x}{x^{3/2}} + \frac{1}{x^{3/2}}$$

$$= 3x^{5/2} + 2x^{1/2} - 7x^{-1/2} + x^{-3/2}$$

$$f'(x) = 5/2(3)x^{3/2} + 2\left(\tfrac{1}{2}\right)x^{-1/2} - 7\left(-\tfrac{1}{2}\right)x^{-3/2} + \left(\tfrac{-3}{2}\right)x^{-5/2}$$

$$= \boxed{15/2\,x^{3/2} + \frac{1}{\sqrt{x}} + \frac{7/2}{x\sqrt{x}} - \frac{3/2}{x^2\sqrt{x}}}$$

6.a) After 2 hours, $1/4(2)^2 = 1$, gives us 1 inch of rainfall. So, the average $= 1/2$ (hours)
$\Rightarrow \boxed{1/2}$ inches per hour.

b) How fast the rain is falling (velocity) indicates that we are looking at a derivative/slope.

$(1/4(t)^2)' = 1/4(2)t = 1/2 t.$

So after 2 hours, velocity is $1/2(2) = \boxed{1}$ inch/hour.

7) $f(2)=5 \quad g(2)=7$
$f'(2)=-1 \quad g'(2)=2$

$(f \cdot g)'(2) = f'(2)g(2) + f(2)g'(2)$
$= (-1)(7) + (5)(2) = -7+10 = \boxed{3}$

$$(f/g)'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2}$$
$$= \frac{(-1)(7) - (5)(2)}{(7)^2} = \boxed{\frac{-17}{49}}$$

8.a) It is not possible to cancel $(2x^2-3)$ in this fraction because we cannot factor out a $(2x^2-3)$ from the numerator, since it is not in both terms.

b) $$f'(x) = \frac{8x^5 - 12x^3 - 4x^5}{(2x^2-3)^2} = \frac{4x^5 - 12x^3}{(2x^2-3)^2} = \boxed{\frac{4x^3(x^2-3)}{(2x^2-3)^2}}$$

c) (From hint) $0 = 4x^3(x^2-3)$
This is true when $4x^3 = 0$ OR $x^2-3=0$
$\Rightarrow f'(x) = 0$ when $x=0$ OR $x = \pm\sqrt{3}$.

d) For this, plug the x-values we found in (c) into the original equation:

$$f(0) = \frac{(0)^4}{2(0)^2-3} = 0, \quad f(\sqrt{3}) = \frac{(\sqrt{3})^4}{2(\sqrt{3})^2-3} = \frac{9}{6-3} = 3$$

$$f(-\sqrt{3}) = \frac{(-\sqrt{3})^4}{2(-\sqrt{3})^2-3} = \frac{9}{3} = 3$$

so, $\boxed{(0,0), (\sqrt{3},3), (-\sqrt{3},3)}$

9) Remember that a horizontal line means that the slope is 0.
So, again like #8, we want $f'(x)=0$.

$$y'(x) = \frac{(x)'(x^2+1) - (x)(x^2+1)'}{(x^2+1)^2} = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

So, we want to find $(1-x^2)=0$

$$\Rightarrow (1-x)(1+x)=0$$

We can have $(1-x)=0$ or $(1+x)=0$, so $x=\underline{1,-1}$ give us 0.

$$y(1) = \frac{1}{1^2+1} = \frac{1}{2} \qquad y(-1) = \frac{-1}{(-1)^2+1} = \frac{-1}{2}$$

So the points are $\boxed{(1, \frac{1}{2})}$ and $\boxed{(-1, -1/2)}$