- Margaret and I will be available to help you with the problems. You should also ask your group members questions, and share your ideas with each other.
- Focus on **understanding** the solution each problem, and on being able to **explain** them to each other.

Recall the statements of the product rule and the quotient rule below (written in "Newton notation" rather than Leibniz notation this time).

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x) \qquad \left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

1. On Worksheet 3, you found the derivative of $f(x) = \frac{x+1}{x-1}$ using the limit definition. Compute this derivative again, this time using the differentiaion rules covered in the last couple days of class.

2. Using your answer to problem 1, find the equation of the tangent line to the curve $y = \frac{x+1}{x-1}$ the point where $x = 3$.

3. We will see a formula later this week that will allow us to differentiate functions like $\sqrt{4x+1}$ (and other functions given by square roots of other functions). For now, you may simply use the following fact.
$$\frac{d}{dx}\sqrt{4x+1} = \frac{2}{\sqrt{4x+1}}$$
Using this fact, together with the product rule, compute the derivative of the function
$$f(x) = (x^2+7)\sqrt{4x+1}.$$
Try to express your answer as a single fraction if possible.

4. Compute $\frac{d}{dx}\left(\frac{2x+1}{3x+1}\right)$ using the quotient rule, and simplify your answer.

5. Compute the derivative of the function
$$f(x) = \frac{3x^4 + 2x^2 - 7x + 1}{x\sqrt{x}}.$$
Try to do this without using the quotient rule.

6. A rainstorm is increasing in intensity as time goes on. A rain gauge measures that the amount of rainfall at time t is $\frac{1}{4}t^2$ inches, where t is the time (in hours) since the start of the storm. For example, after 2 hours there has been 1 inch of rainfall, and after 4 hours there have been 4 inches of rainfall.
   (a) What was the *average* rate of rainfall (in inches per hour) during the first two hours of the storm?
   (b) How fast was the rain falling at exactly 2 hours after the beginning of the storm?

7. Suppose that $f(x)$ and $g(x)$ are two functions, satisfying the following four equations at $x = 2$.

$$\begin{aligned} f(2) &= 5 \qquad & g(2) &= 7 \\ f'(2) &= -1 \qquad & g'(2) &= 2 \end{aligned}$$

   Evaluate $(f \cdot g)'(2)$ and $(f/g)'(2)$.

8. Consider $f(x) = \dfrac{x^4}{2x^2 - 3}$. Applying the quotient rule to this function, but not yet simplifying, gives the following (check this for yourself).

$$f'(x) = \frac{(4x^3)(2x^2 - 3) - (x^4)(4x)}{(2x^2 - 3)^2}$$

   (a) Explain why it is not possible to cancel $2x^2 - 3$ in this fraction.

   (b) Simplify this expression, factoring the numerator as much as possible.

   (c) Using your answer in (b), solve the equation $f'(x) = 0$. It is useful to know that a fraction is equal to 0 when its numerator is equal to 0.

   (d) Find all points on the curve $y = f(x)$ where the tangent line is horizontal.

9. Find all points on the curve $y = \dfrac{x}{x^2 + 1}$ where the tangent line is horizontal.