$\boxed{\text{Brief solutions}}$ for worksheet 7.

(1)

$$f(x) = x\sqrt{1-x}$$

$$f'(x) = 1\cdot\sqrt{1-x} + x\cdot\frac{1}{2\sqrt{1-x}}\cdot(-1)$$

$$= \frac{\sqrt{1-x}}{1}\cdot\frac{2\sqrt{1-x}}{2\sqrt{1-x}} - \frac{x}{2\sqrt{1-x}}$$

$$= \frac{2-2x-x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}$$

undef. at $\boxed{x=1}$

& zero at $\boxed{x=2/3}$

(two critical numbers).

(2)

$$f(x) = \frac{x^2+1}{x+3}$$

$$f'(x) = \frac{2x(x+3)-(x^2+1)\cdot 1}{(x+3)^2}$$

$$= \frac{2x^2+6x-x^2-1}{(x+3)^2}$$

$$= \frac{x^2+6x-1}{(x+3)^2}$$

undef. @ $x=-3$, but this isn't in f's domain

$\Rightarrow$ not a critical pt.

zero when $x^2+6x-1=0$

ie. $x = \frac{-6\pm\sqrt{36+4}}{2}$

$= \boxed{-3\pm\sqrt{10}}$.

(3)

$$f(x) = x^{3/4} - 2x^{1/4}$$

$$f'(x) = \frac{3}{4}x^{-1/4} - \frac{1}{2}x^{-3/4}$$

undef. at $\boxed{x=0}$ (divis. by 0)

equal to 0 when $\frac{3}{4}x^{-1/4} = \frac{1}{2}x^{-3/4} \iff x^{1/2} = \frac{2}{3}$

$\iff \boxed{x = 4/9}$

(4)

$F(x) = x^3-3x^2$ on $[-1,1]$

$F'(x) = 3x^2-6x$

$= 3x(x-2)$

(no undefined values)

$F'(x)=0$ @ $x=0$ & $x=2$,

but $x=2$ is outside $[-1,1]$.

check $x=-1, 1, \& 0$:

$F(-1) = -4 \leftarrow$ min

$F(1) = -2$

$F(0) = 0 \leftarrow$ max.

max value :	0	@ $x=0$
min value:	-4	@ $x=-1$.

⑤

$G(x) = (x-1)^2(x-9)^2$ on $[0,8]$

$G'(x) = 2(x-1)(x-9)^2 + (x-1)^2 2(x-9)$

$= 2(x-1)(x-9)\cdot[(x-9)+(x-1)]$

$= 2(x-1)(x-9)(2x-10)$

$= 4(x-1)(x-5)(x-9)$

$G'(x)=0 \iff x=\underline{1, 5}$, or 9

but $9 \notin [0,8]$.

(& defined everywhere).

check 0, 8, 1 & 5.

$G(0) = 1^2 9^2 = 81$

$G(8) = 7^2 \cdot 2^2 = 196$

$G(1) = 0$ ← min

$G(5) = 4^2 \cdot 4^2 = 256$ ← max

$\boxed{\text{max value } 256 \text{ @ } x=5 \quad \text{min value } 0 \text{ @ } x=1.}$

⑥

$H(x) = \dfrac{10x}{x^2+1}$ on $[0,2]$

$H'(x) = \dfrac{10\cdot(x^2+1) - 10x\cdot 2x}{(x^2+1)^2} = \dfrac{10x^2+10-20x^2}{(x^2+1)^2} = \dfrac{10-10x^2}{(x^2+1)^2}$

$= \dfrac{10(1-x^2)}{(x^2+1)^2}$.

Defined everywhere since $x^2+1 \neq 0$.

$H'(x)=0 \iff x^2=1 \iff x=\pm 1$. only $x=+1$ is in $[0,2]$.

Check 0, 1, & 2:

$H(0) = 0$ ← min

$H(1) = \frac{10}{2} = 5$ ← max

$H(2) = 20/5 = 4$

$\boxed{\text{max value } 5 \text{ @ } x=1 \quad \text{min value } 0 \text{ @ } x=0}$

⑦

$f(x) = x(x^2-5)^2$ on $[-2,2]$

$f'(x) = 1\cdot(x^2-5)^2 + x\cdot 2(x^2-5)\cdot 2x$

$= (x^2-5)\cdot[(x^2-5)+4x^2]$

$= (x^2-5)(5x^2-5)$

$= 5(x^2-5)(x^2-1)$

$f'(x)$ always defined.

$f'(x)=0 \iff x=\pm 1$ or $\pm\sqrt{5}$

only ± 1 are in $[-2,2]$.

$f(-2) = -2\cdot 1 = -2$

$f(-1) = -1\cdot 4^2 = -16$ ← min

$f(1) = 1\cdot 4^2 = 16$ ← max

$f(2) = 2\cdot 1 = 2$

$\boxed{\text{max value } 16 \text{ @ } x=1 \quad \text{min value } -16 \text{ @ } x=-1.}$