- Margaret and I will be available to help you with the problems. You should also ask your group members questions, and share your ideas with each other.
- Focus on **understanding** the solution each problem, and on being able to **explain** them to each other.

Optimization problems

Refer to the reverse side for some tips on optimization problems

1. Among all rectangles with perimeter 20, find the one with maximum area.
2. A farmer wishes to construct a rectangular pen of area 50 square meters. One side of the pen will be the side of a barn (so there is no need to build fencing on this side), while the other three sides will need to be built out of fencing. What dimensions minimize the amount of fencing required?
3. We wish to construct a cardboard box of volume 16 cubic feet. The box will have a square base, and both the top and bottom are a double layer of cardboard, while the sides will only be a single layer. What dimensions will minimize the amount of cardboard required?

   Note: we began this problem in class yesterday; try to reconstruct the first several steps yourself, then check your notes to compare.
4. Find two positive numbers with product 18 and such that the sum of one plus twice the other is as small as possible.

Review problems (from early in the course)

5. Evaluate the following limits, and justify your answers.

   (a) $\lim_{x\to -2} \dfrac{x^2+3x+2}{x^2+x-2}$

   (b) $\lim_{x\to 5} \dfrac{25-x^2}{\sqrt{x+4}-3}$

   (c) $\lim_{x\to 7^+} \dfrac{|7-x|}{x^2-x-42}$

   (d) $\lim_{x\to -5} \dfrac{\frac{5}{x}-\frac{1}{x+4}}{x+5}$
6. Evaluate the **derivative** of each function, but do not simplify your answer.

   (a) $\dfrac{\sqrt{x^3-x^{-8}}}{(x^2+5)^4}$

   (b) $\left(\dfrac{1}{x}-\dfrac{1}{x^4}\right)\sqrt{x^2+1}$
7. Let $f(x)=\dfrac{x}{x+2}$. Calculate $f'(x)$ using the **limit definition** of the derivative. Check your answer using the quotient rule.

Tips and a Systematic Procedure for Solving Optimization (Applied Max-Min) Problems

Math 111

- Read and understand the problem.

- **Diagram:** Draw and label the diagram for the problem. (Try to imagine a few extreme cases here in order to give you a sense of the possible options.)

- **Variables:** Assign appropriate variables to key quantities. Write them down.

- **Equation(s) relating variables:**

  - Write an equation relating the variables based on the given fixed information. This is the equation that represents some restricted relationship between the variables.

  - Write an equation for the quantity to be maximized or minimized in terms of one or more of the variables.

  - Use the first fixed equation, representing the given restricted conditions of the problem, to eliminate all but one of these variables in the second equation and thereby express the desired quantity as a function of a single variable.

  - Determine the set of possible values for this single variable. These values can often determine a closed interval for the function's domain; we can think about them as *common sense choices* for the variable.

- **Maximize/Minimize:** Use calculus techniques and max-min theory to find the desired absolute maximum or minimum value.

  - Differentiate and find the critical numbers (where the quantity's derivative equals zero or is undefined).

  - Identify the extrema. You can use sign-testing and the First Derivative Test for Absolute Extreme Values in Section 3.7, or use the Closed Interval Method if you have a closed interval for the domain. Make sure that you actually investigate whether you have an absolute maximum or minimum value.

- **Answer:** Answer the question posed in the original problem. Interpret your results and give a detailed answer to the orginal question. Check if your answer satisfies the conditions in the initial word problem.