Practice Final A

1. Compute the following derivatives.

a. $\dfrac{d}{dx}[e^x \sin(5x)]$

b. $\dfrac{dy}{dx}$, where $e^{2x} - e^y + xy = e^2$.

c. $G'(\ln(3))$, where $G(x) = h(e^x)$ and it is known that $h(3) = 2$ and $h'(3) = -5$.

d. $g''(x)$, where $g(x) = \sqrt{x}\ln x + \ln(\sqrt{x})$.

2. Calculate the following limits.

a. $\displaystyle\lim_{x\to -1} \frac{x^2+4x+3}{2x^2+3x+1}$

b. $\displaystyle\lim_{x\to 9} \frac{3-\sqrt{x}}{x-9}$

c. $\displaystyle\lim_{x\to 1} \frac{2f(x)-4x}{f(2x)-5}$, where $f(x) = x^2+1$

d. $\displaystyle\lim_{x\to 2^-} \frac{|x-2|}{x^2-4}$

3. Compute the following integrals.

a. $\displaystyle\int \frac{(x+2)^2}{x}\,dx$

b. $\displaystyle\int_{\pi/12}^{\pi/6} \sec^2(2x)\,dx$

c. $\displaystyle\int \frac{5}{3x+2}\,dx$

d. $\displaystyle\int_1^e \frac{1}{x}\cos\left(\frac{\pi}{4}\ln x\right)dx$

e. $\displaystyle\int_0^3 |x-1|\,dx$ (*Hint*: cut the interval into two pieces and do each piece separately.)

4. Let $f(x) = \dfrac{4}{x+3}$. Calculate $f'(1)$ using the **limit definition of the derivative**.

5. Find an equation for the tangent line to the graph of $y = \ln(x^2+1)$ at the point where $x = 2$.

6. A ladder 5 meters long is leaning against a vertical wall. The base of the ladder starts to slide away from the wall along the (horizontal) ground, and so the top of the ladder starts to slide down the wall. At the moment when the top of the ladder is 4 meters above the ground, it is sliding down the wall at 1 meter per second. How fast is the angle between the ladder and the ground increasing (or decreasing) at that moment?

7. Find the absolute maximum and absolute minimum values of the function

$$g(x) = (x^2-3)e^x$$

on the interval $[0,4]$.

8. Let $F(x) = 3x^4 + 2x^3 - 3x^2 - 5$. Find all of the critical numbers of F, and classify each of them as local maximum, local minimum, or neither.

9. Let $f(x) = \dfrac{3x^3 + 9x^2 + 10x}{(x+1)^3}$. Take my word for it that

$$f'(x) = \frac{-2(x-5)}{(x+1)^4}, \qquad \text{and} \qquad f''(x) = \frac{6(x-7)}{(x+1)^5}.$$

Sketch the graph of $y = f(x)$, clearly indicating **horizontal and vertical asymptotes**, **local extrema**, **inflection points**, and **intervals of increase and decrease and of concavity**. Please do **NOT** try to draw your graph to scale.

10. A farmer needs to fence off a rectangular field of area of $2000\,\mathrm{m}^2$ and then divide the rectangle into two pens with an extra middle fence running parallel to two of the sides. The outside fencing costs \$20 per meter, while the middle fencing costs \$10 per meter. What should the dimensions of the field be to minimize the cost of the fence?

11. Compute the integral $\displaystyle\int_0^3 \left(x^2 - 1\right)\, dx$ directly from the definition, i.e., **as a limit of Riemann sums**.

12. Find a function $f(x)$ such that $f'(x) = \dfrac{x^2 - 1}{x}$ with $f(1) = 2$.