Practice Final B

1. Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 2} \frac{(x+1)^2-9}{x^2+4}$ (b) $\displaystyle\lim_{x\to 3^-} \frac{x^2-8x+15}{1-8x+g(x+1)}$, where $g(x) = x^2+7$.

(c) $\displaystyle\lim_{x\to 8} \frac{8-x}{\sqrt{x+1}-3}$ (d) $\displaystyle\lim_{x\to 6} \frac{x^2-4x-12}{|6-x|}$

2. Compute each of the following derivatives. Simplify numerical answers. Do not simplify your algebraically complicated answers.

(a) $f'\left(\frac{\pi}{12}\right)$, where $f(x) = \sec^2(2x) + \sin(4x)$ (b) $\displaystyle\frac{d}{dx} \ln\left(\frac{(x^2+1)^{4/7}\, e^{\tan x}}{\sqrt{1+\sqrt{x}}}\right)$

(c) $g'(x)$, where $g(x) = e^{\sqrt{x^2+7\cos x}} + \dfrac{1}{\sqrt{e^{x^2+7\cos x}}}$ (d) $\dfrac{dy}{dx}$, if $e^{xy^3} + \sin^3 x = \ln(xy) + \sin(e^9)$.

3. Compute each of the following integrals.

(a) $\displaystyle\int_{\pi/18}^{\pi/9} \tan(3x)\, dx$ (b) $\displaystyle\int \frac{\left(x^{7/2}+1\right)^2}{\sqrt{x}}\, dx$ (c) $\displaystyle\int_e^{e^4} \frac{3}{x\sqrt{\ln x}}\, dx$ (d) $\displaystyle\int \frac{1}{x^2\, e^{1/x}}\, dx$

4. Let $f(x) = \dfrac{x^2+1}{x-3}$. Calculate $f'(x)$ in two different ways:

(a) Using the Quotient Rule.

(b) Using the **limit definition** of the derivative.

5. Compute $\displaystyle\int_1^9 (x-4)\, dx$ using each of the following **three** different methods:

(a) Area interpretations of the definite integral,

(b) Fundamental Theorem of Calculus,

(c) Riemann Sums and the limit definition of the definite integral.

6. Find the equation of the tangent line to $y = \cos(\ln(x+1)) + \ln(\cos x) + e^{\sin x} + \sin(e^x - 1)$ at the point where $x = 0$.

7. Let $\boldsymbol{f(x) = \dfrac{x}{e^x} = xe^{-x}}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

Take my word that $\displaystyle\lim_{x\to\infty} f(x) = 0$ and $\displaystyle\lim_{x\to-\infty} f(x) = -\infty$.

8.

(a) State the definition of **continuity** of $f(x)$ **at** $x = a$.

(b) Carefully sketch the graphs of $y = e^x$ and $y = \ln x$.

(c) Use the definition of continuity from part (a) to find which k-value makes $f(x)$ continuous at

$x = 1$ for $$f(x) = \begin{cases} e^x & \text{if } x \leq 1 \\ \ln x + k & \text{if } x > 1 \end{cases}$$

(d) Using your k value from part (c) above, sketch $f(x)$. Is this piece-wise defined function $f(x)$ continuous on $(-\infty, \infty)$? Explain.

9. A state trooper is parked 30 meters east of a road that runs north-south. He spots a speeding car and (using his radar gun) determines that the car's distance to him is decreasing at 32 meters per second at the moment when the car is at a point 50 meters from him. (That is, 50 meters along the diagonal from him to the car.) How fast is the car actually going at that moment?

10. A rectangular poster is to contain $50\,\text{in}^2$ of printed matter with margins of 4 inches at each of the top and bottom, and margins of 2 inches on each side. What are the height and width of the poster fitting those requirements that has the smallest possible area?

11. Find the absolute maximum and minimum values of the function

$$g(x) = 3x - x \ln x$$

on the interval $[1, e^4]$.

12. Consider an object moving on the number line such that its velocity at time t seconds is $\boldsymbol{v(t) = 4 - t^2}$ feet per second. Also assume that the position of the object at one second is $\frac{5}{3}$.

(a) Compute the acceleration function $a(t)$.

(b) Compute the position function $s(t)$.