Practice Test A for Midterm Exam 1

This practice exam is a slightly modified version of an exam written by Rob Benedetto.

Instructions: This optional exam is for practice, to give you an idea of what our in-class midterm exam will be like. I'd recommend that you try taking it in exam conditions: 50 minutes, closed-book, with access to your one-page note sheet (front and back).

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1. [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(**a**) $\lim_{x\to -3} \dfrac{x^2-2x-15}{x^2+x-6} =$ (**b**) $\lim_{x\to 5} \dfrac{x^2-2x-15}{|5-x|} =$

(**c**) $\lim_{x\to 2} \dfrac{x^2-2x-15}{x^2+x-6} =$ (**d**) $\lim_{x\to 5} \dfrac{x^2-2x-15}{x^2+x-6} =$

(**e**) $\lim_{x\to 2} \dfrac{x+7}{(x-2)^2} =$ (**f**) $\lim_{x\to -1} \dfrac{H(x+1)-H(-1-x)}{x+1} =$ where $H(x)=\sqrt{x+2}$

2. [13 Points] Use translation, etc. to graph the following two functions:

$$f(x) = 5 + \sqrt{6-x} \qquad\qquad g(x) = \frac{1}{10}(x+2)^2$$

3. [15 Points] Suppose that $f(x) = \dfrac{x+7}{x-3}$. Compute $f'(x)$ using the **limit definition of the derivative**.

4. [10 Points] Suppose that $f(x) = x^2 - 7x - 12$. Write the **equation of the tangent line** to the curve $y = f(x)$ when $x = -2$. **Use the limit definition of the derivative when computing the derivative.**

5. [12 Points] Suppose that f and g are functions, **and**

- $\lim_{x\to 7} f(x) = 5$
- $\lim_{x\to 7} g(x) = -3$
- $f(5) = 7$
- $g(7) = \lim_{x\to 7} g(x)$

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

(a) $\lim_{x\to 7} \sqrt{3f(x) - 7g(x)} =$

(b) $\lim_{x\to 7} \dfrac{f(x)}{1-x} =$

(c) $g \circ f(5) =$

(problems continue next page)

6. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 3 \\ 1 & \text{if } x = 3 \\ 6 - 2x & \text{if } 0 < x < 3 \\ 16 - x^2 & \text{if } -4 < x \leq 0 \\ \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

(c) Compute $\begin{cases} \lim\limits_{x \to 0^+} f(x) = \\ \lim\limits_{x \to 0^-} f(x) = \\ \lim\limits_{x \to 0} f(x) = \end{cases}$

(d) Compute $\begin{cases} \lim\limits_{x \to 3^+} f(x) = \\ \lim\limits_{x \to 3^-} f(x) = \\ \lim\limits_{x \to 3} f(x) = \end{cases}$

(e) Compute $\begin{cases} \lim\limits_{x \to -4^+} f(x) = \\ \lim\limits_{x \to -4^-} f(x) = \\ \lim\limits_{x \to -4} f(x) = \end{cases}$