Solutions to Practice Test A for Midterm Exam 2

1. **(36 points)** Compute the following derivatives by any legal method.

(a). $f'(x)$, where $f(x) = \tan(5x^2 - 8)$.

(b). $\frac{d}{dt}\Big((1-t^4)\sqrt{\cos t}\Big)$.

(c). y', where $xy + y^3 = 4x^2$.

(d). $g'(x)$, where $g(x) = \frac{x^2+3x}{x+1}$.

(e). $h''(x)$, where $h(x) = \frac{x^3+4}{\sqrt{x}}$.

Solutions. (a) $f'(x) = \sec^2(5x^2-8) \cdot 10x = \boxed{10x\sec^2(5x^2-8)}$

(b) The derivative is $-4t^3(\cos t)^{1/2} + (1-t^4)\cdot\frac{1}{2}\cdot(\cos t)^{-1/2}\cdot(-\sin t)$

$$= \boxed{-(\cos t)^{-1/2}\left[4t^3\cos t + \frac{1}{2}(1-t^4)\sin t\right]}$$

(c) Implicit Diff: $y + xy' + 3y^2y' = 8x$, so $(x+3y^2)y; = 8x - y$, so $\boxed{y' = \frac{8x-y}{x+3y^2}}$

(d) $g'(x) = \frac{(2x+3)(x+1)-(x^2+3x)\cdot 1}{(x+1)^2} = \frac{2x^2+5x+3-x^2-3x}{(x+1)^2} = \boxed{\frac{x^2+2x+3}{(x+1)^2}}$

(e) We have $h(x) = x^{-1/2}(x^3+4) = x^{5/2} + 4x^{-1/2}$, so $h'(x) = \frac{5}{2}x^{3/2} - 2x^{-3/2}$, and hence $h''(x) = \boxed{\frac{15}{4}x^{1/2} + 3x^{-5/2}}$

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2. **(14 points)** Suppose f, g, h are functions such that

$$f(2) = 4, \quad f'(2) = -3, \quad g(1) = 2, \quad g'(1) = 5, \quad h(1) = 7, \quad h'(1) = -2.$$

Let $F(x) = f(g(x))$ and $G(x) = g(x)\cdot h(x)$. Compute $F'(1)$ and $G'(1)$.

Solutions. $F'(x) = f'(g(x))\cdot g'(x)$, so $F'(1) = f'(g(1))\cdot g'(1) = f'(2)\cdot 5 = (-3)\cdot 5 = \boxed{-15}$

$G'(x) = g'(x)h(x) + g(x)h'(x)$, so $G'(1) = g'(1)h(1) + g(1)h'(1) = (5)(7) + (2)(-2) = 35 - 4 = \boxed{31}$

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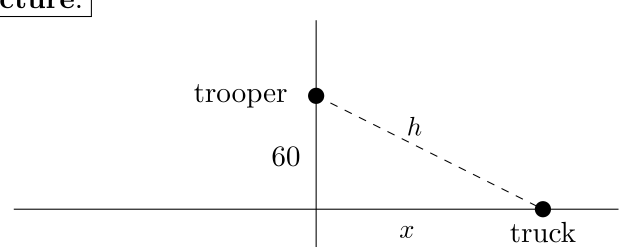
3. **(20 points)** A state trooper is parked on a North-South road 60 meters from where it intersects an East-West road. Meanwhile, a truck is driving along the East-West road. At the moment the truck is 80 meters past the intersection, the trooper (using his radar gun) sees that the truck's distance from him is increasing at 12 m/sec. How fast is the truck actually going at that time?

Solution. Here's the **Picture**:



$\boxed{\textbf{Variables}:}$

x =East-West distance from truck to point on road, in m
h =diagonal distance from trooper to truck, in m

(And t =time, in sec)

Main $\boxed{\textbf{Equation}:}$ $x^2 + 60^2 = h^2$

$\boxed{\textbf{Differentiate}}$ (implicitly, w.r.t. time): $2x\dfrac{dx}{dt} = 2h\dfrac{dh}{dt}$

$\boxed{\textbf{Use key moment info}:}$

At the key moment, we have $x = 80$ m and we are told that $\dfrac{dh}{dt} = 12\text{m/sec}$

Also, plugging $x = 80$ into the original equation gives $80^2 + 60^2 = h^2$,
i.e., $h^2 = 6400 + 3600$, i.e., $h^2 = 10000$, so $h = \pm 100$. Must be $h = 100\,\text{m}$.

Plugging these values into the derivative equation above,
we have $2(80)\dfrac{dx}{dt} = 2(100)(12)$, i.e., $\dfrac{dx}{dt} = \dfrac{1200}{80} = \dfrac{30}{2} = \boxed{15\,\text{m/sec}}$

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4. **(18 points)** Let $g(x) = \dfrac{x+4}{x^2+9}$.
Find the absolute minimum and absolute maximum values of g on the interval $[-4, 4]$.
Solution. (By Closed Interval Method):

$$g'(x) = \frac{1(x^2+9) - (x+4)(2x)}{(x^2+9)^2} = \frac{x^2+9-2x^2-8x}{(x^2+9)^2} = \frac{-x^2-8x+9}{(x^2+9)^2} = \frac{-(x+9)(x-1)}{(x^2+9)^2},$$

which is **always defined**.

Solving $g' = 0$ gives $x = -9, 1$, but -9 is not in the interval.

So the only critical number is $x = 1$. Testing it and endpoints:

$$g(-4) = \frac{0}{16+9} = 0, \quad g(1) = \frac{5}{1+9} = \frac{1}{2}, \quad g(4) = \frac{8}{16+9} = \frac{8}{25}$$

So the $\boxed{\text{absolute maximum is } \frac{1}{2}}$ and the $\boxed{\text{absolute minimum is } 0}$

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5. **(12 points)** Let $f(x) = \sin^3(4x) + \sec(4x) - 8\sin(2x)$. Compute $f'\left(\dfrac{\pi}{12}\right)$. Simplify.

Solution. $f'(x) = 3\sin^2(4x)\cdot\cos(4x)\cdot 4 + \sec(4x)\tan(4x)\cdot 4 - 8\cos(2x)\cdot 2$

$= 12\sin^2(4x)\cos(4x) + 4\sec(4x)\tan(4x) - 16\cos(2x)$,

$$\text{So } f'\left(\frac{\pi}{12}\right) = 12\sin^2\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) + \frac{4\sin\left(\frac{\pi}{3}\right)}{\cos^2\left(\frac{\pi}{3}\right)} - 16\cos\left(\frac{\pi}{6}\right) = 12\left(\frac{\sqrt{3}}{2}\right)^2\cdot\frac{1}{2} + \frac{4(\sqrt{3}/2)}{(1/2)^2} - 16\left(\frac{\sqrt{3}}{2}\right)$$

$$= 6\left(\frac{3}{4}\right) + 8\sqrt{3} - 8\sqrt{3} = \boxed{\frac{9}{2}}$$