Practice Test A for Midterm Exam 3

Instructions: This optional exam is for practice, to give you an idea of what our in-class midterm exam will be like. I'd recommend that you try taking it in exam conditions: 50 minutes, closed-book.

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1. **(20 points)** You need to construct a box with a square base with a fixed volume of 24 cubic feet. The material for the bottom and top costs \$3 per square foot, and the material for the sides costs \$1 per square foot. What are the **dimensions** that minimize the cost required to build such a box? What is that **minimum cost**?

2. **(25 points)** Compute $\int_1^3 x^2 - 3x \, dx$ using each of the following **two** different methods:

(a) The Fundamental Theorem of Calculus.

(b) Riemann Sums and the limit definition of the definite integral.
Note: I will not ask you to do a full-scale integral via the limit definition on a 50 minute exam, but it is still good practice, so I've left it on this practice exam. -NP (2023)

3. **(30 points)** Let $f(x) = \dfrac{-x^2+x+2}{x^2-2x+1} = \dfrac{-x^2+x+2}{(x-1)^2}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve $y = f(x)$.

You may take my word for it that:

$$f'(x) = \frac{x-5}{(x-1)^3} \qquad \text{and} \qquad f''(x) = \frac{-2x+14}{(x-1)^4}$$

4. **(10 points)** Compute the following limits.

(a) $\lim\limits_{x\to\infty} \dfrac{x^2-x+5}{3x^7+x^6-2022}$ (b) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{4x^2+5x}}{6x+7}$

5. **(15 points)** Compute the following definite and indefinite integrals.

(a) $\displaystyle\int \frac{y^2-4y+1}{\sqrt{y}}\, dy$ (b) $\displaystyle\int_{-\pi/4}^{5\pi/6} 3x + \cos x \, dx$