Math 111 Gratefully borrowed from Professor Rob Benedetto

Practice Problems for Midterm Exam 3

Instructions: The point of this set of practice problems is **NOT** that you should plan to do all of these problems. Instead, the point is that you should skip around and try various different types of problems. And if you find you could use more practice with a particular type of problem, you should be able to find several more like it here.

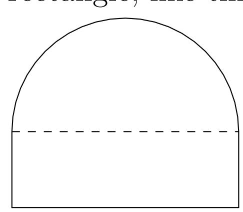
So don't try to do all of these problems. But try to do a lot of them — a broad variety of them, but also extras on any particular topics that you find you could use the most practice on.

Optimization

1. Show that of all rectangles with area 64, the one with the smallest perimeter is a square.

2. A rectangle lies in the first quadrant, with one vertex at the origin, two sides along the coordinate axes, and the fourth vertex on the line $x + 2y - 6 = 0$. Find the maximum area of the rectangle.

3. A Norman window is a window in the shape of a rectangle with a semicircle on top of it, with diameter the same length as the rectangle, like this:



Suppose a Norman window is to have a perimeter of 30 ft. What dimensions will make the area the largest?

4. A forest ranger can walk through the woods at a rate of 1 m/sec and along a road at 2 m/sec. The ranger is in the woods 100m from the nearest point on a straight road, and he wants to get to a car stopped 300m further down the road. What path should he take to get to the car in the shortest time?

5. In the forest ranger problem, what is the best path if the car is only 40m down the road instead of 300m?

6. We need a cardboard box with a square base, open top, and a vertical partition (wall) inside, parallel to one of the sides. (The partition is also made of cardboard.) If the total volume of the box needs to be $50\,\text{ft}^3$, what dimensions will use the least amount of cardboard?

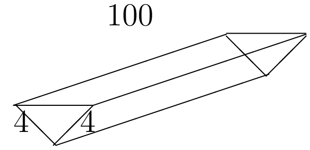
7. Consider a cone such that the height is 6 inches high and its base has diameter 6 in. Inside this cone we inscribe a cylinder whose base lies on the base of the cone and whose top intersects the cone in a circle. What is the maximum volume of the cylinder?

8. A poster is to have an area of 180 square inches, with margins of 1 inch at the bottom and sides, and a margin of 2-inches at the top. What dimensions will give the largest printed area?

9. A toolshed with a square base and a flat roof is to have volume of 800 cubic feet. If the material for the floor costs \$6 per square foot, the roof \$2 per square foot, and the sides \$5 per square foot, determine the dimensions of the most economical shed.

10. A manufacturer wishes to produce rectangular containers with square bottoms and tops, each container having a capacity of 250 cubic inches. The material for the sides costs \$2 per square inch for the sides, and the material for the top and bottom costs \$4 per square inch. What dimensions of the containers will minimize the cost?

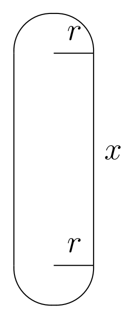
11. A rectangular sheet of metal 8 inches wide and 100 inches long is folded along the center to form a triangular trough. Two triangular pieces of metal are attached to the ends of the trough, and the trough is to be filled with water:



(a). How deep should the trough be to maximize the capacity of the trough?

(b). What is this maximum possible capacity?

12. An outdoor track is to be created in the shape shown and is to have perimeter of 440 yards. Find the dimensions for the track that maximize the area of the rectangular portion of the field enclosed by the track.



13. Show that the entire region enclosed by the outdoor track in the previous problem has maximum area if the track is circular.

Antiderivatives / Initial value problems

14. Find a function $f(x)$ that satisfies $f''(x) = 12x^2 + 5$ and $f'(1) = 5$, and which passes through the point $(1, 3)$.

15. Find a function $f(x)$ that satisfies $f''(x) = x + \sin x$, $\quad f'(0) = 6$, and $\quad f(0) = 4$.

16. Find a function $f(x)$ that satisfies $f''(x) = 2 - 12x$, $\quad f(0) = 9$, and $\quad f(2) = 15$.

17. Find a function $f(x)$ that satisfies $f''(x) = 20x^3 + 12x^2 + 4$, $\quad f(0) = 8$ and $\quad f(1) = 5$.

Curve Sketching

For each of the following functions, discuss domain, vertical and horizontal asymptotes, intervals of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

18. $f(x) = x^3 - 3x^2 + 3x + 10$
19. $f(x) = \dfrac{3x^2}{1 - x^2}$
20. $f(x) = \dfrac{x}{x - 2}$
21. $f(x) = 2x^3 + 5x^2 - 4x$
22. $f(x) = 3x^4 + 4x^3$
23. $f(x) = x^4 - 6x^2$
24. $f(x) = \dfrac{3x^5 - 20x^3}{32}$
25. $f(x) = \dfrac{1}{x^2 - 9}$

26. $f(x) = \dfrac{2x^3 + 45x^2 + 315x + 600}{x^3}$. And **before you start**:
take my word for it $f'(x) = \dfrac{-45(x+4)(x+10)}{x^4}$ and $f''(x) = \dfrac{90(x+5)(x+16)}{x^5}$.
(So do NOT re-compute those derivatives!)

Limits at infinity

27. Compute $\displaystyle\lim_{x\to\infty} \frac{x^3 + 1}{x^7 + 2x^{7/2}}$
28. Compute $\displaystyle\lim_{x\to\infty} \frac{x^6 + 1}{x^3 + 9x^2 + 7}$
29. Compute $\displaystyle\lim_{x\to\infty} \frac{2x + 1}{\sqrt{9x^2 + 5}}$
30. Compute $\displaystyle\lim_{x\to-\infty} \frac{2x + 1}{\sqrt{9x^2 + 5}}$

Riemann sums

31. Use Riemann Sums to estimate $\displaystyle\int_0^1 x^2 + 1\, dx$ using 4 equal-length subintervals and right endpoints.

32. Compute $\displaystyle\int_1^4 x - 1\, dx$ using three different methods:
(a) using the Area interpretation of the definite integral.
(b) using the Fundamental Theorem of Calculus.
(c) from the limit definition, i.e., using Riemann Sums.

33. Evaluate $\displaystyle\int_{-1}^1 x\, dx$ using Riemann Sums.

34. Evaluate $\displaystyle\int_0^2 x^2 - 5x\, dx$ using Riemann Sums.

Integrals

Compute the following definite and indefinite integrals.

35. $\displaystyle\int_{\pi/6}^{\pi/2} \csc^2 t \, dt$

36. $\displaystyle\int \frac{(2w^2+1)(w-3)}{\sqrt{w}} \, dw$

37. $\displaystyle\int x^5 + \frac{1}{x^5} + \sqrt[5]{x} + \frac{x}{5} \, dx$

38. $\displaystyle\int_1^4 \frac{4}{x^3} - \frac{3}{\sqrt{x}} \, dx$

39. $\displaystyle\int (\sec\theta + \tan\theta)\sec\theta \, d\theta$