Math 111-01, Fall 2022 Professor Rob Benedetto

Practice Problems for Midterm Exam 1

Instructions: The point of this set of practice problems is **NOT** that you should plan to do all of these problems; there are **WAY** too many. Instead, the point is that you should skip around and try various different types of problems. And if you find you could use more practice with a particular type of problem, you should be able to find several more like it here.

So don't try to do all of these problems. But try to do a lot of them — a broad variety of them, but also extras on any particular topics that you find you could use the most practice on.

Functions: Please state what the domain is for each of the following functions.

1. $f(x) = \dfrac{x+2}{x-1}$
2. $g(x) = \sqrt{x-2}$
3. $m(x) = \sqrt{2-x}$
4. $G(x) = \dfrac{1}{\sqrt{2-x}}$
5. $h(x) = \dfrac{x-3}{x^2+3}$
6. $W(x) = \dfrac{x^2+6x+8}{x+2}$

Combining Functions:

7. Let $g(x) = \dfrac{x+1}{x}$. Compute (and simplify, if possible) the following:

   (a) $g(t-2) =$ (b) $\dfrac{g(2+h)-g(2)}{h} =$

8. Let $f(x) = \dfrac{1}{x+1} - \dfrac{1}{x}$. Compute (and simplify, if possible) the following:

   (a) $f(t-1) =$ (b) $f\left(\dfrac{1}{t}\right) =$

9. Graph the following functions using scaling, translation, etc.

   (a) $y = 2 + \sqrt{x+1}$ (b) $y = \dfrac{3}{(x-2)^2}$ (c) $y = 2(x-1)^4 - 3$

Limit Practice Problems: Evaluate the following limits. Always justify your work. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

10. $\lim\limits_{w\to 0} \dfrac{16}{w}$
11. $\lim\limits_{t\to 2} \dfrac{3-t}{t-2}$
12. $\lim\limits_{t\to 2} \dfrac{3-t}{(t-2)^2}$
13. $\lim\limits_{x\to 4} \dfrac{(x+2)^2}{x^2-3x-4}$
14. $\lim\limits_{x\to 4} \dfrac{x-4}{x^2-3x-4}$
15. $\lim\limits_{x\to 4} \dfrac{x^2-2x-8}{x^2-3x-4}$
16. $\lim\limits_{x\to 6} \dfrac{x^2-4x-12}{x^2-3x-18}$
17. $\lim\limits_{x\to 1} \dfrac{x^2-4x-12}{x^2-3x-18}$
18. $\lim\limits_{x\to 0} \dfrac{x^2-4x-12}{x^2-3x-18}$
19. $\lim\limits_{x\to -3} \dfrac{x^2-4x-12}{x^2-3x-18}$
20. $\lim\limits_{x\to -2} \dfrac{x^2-4x-12}{x^2-3x-18}$
21. $\lim\limits_{x\to 0} \dfrac{x^2-4x-12}{x^2-7x}$
22. $\lim\limits_{x\to 0} \dfrac{x^2-4x}{x^2-7x}$
23. $\lim\limits_{x\to 3} \dfrac{x^2-9}{|x-3|}$
24. $\lim\limits_{x\to 0} \dfrac{x^3+209x^2+200x}{|x|}$

25. $\lim_{x\to -5} \dfrac{x^2+6x+5}{|x+5|}$

26. $\lim_{t\to -1} \dfrac{200(t^2+6t+5)}{t^2+t}$

27. $\lim_{t\to 1} t^{300}+t^{200}+t^{100}$

28. $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x+3}-2}$

29. $\lim_{x\to 9} \dfrac{9x-x^2}{3-\sqrt{x}}$

30. $\lim_{x\to 1} \dfrac{\sqrt{x^2+8}-3}{x-1}$

31. $\lim_{x\to -4} \dfrac{x^2-3x-28}{x^2+4x}$

32. $\lim_{x\to 0} \dfrac{x^2-3x-28}{x^2+4x}$

33. $\lim_{x\to 3} \dfrac{\dfrac{2}{x+3}-\dfrac{1}{3}}{x-3}$

34. $\lim_{x\to 1} \dfrac{x^2-1}{|x-1|}$

35. $\lim_{x\to -5} \dfrac{x^2+6x+5}{|x+5|}$

36. $\lim_{x\to -1} \dfrac{x^2+3x+2}{(x+1)^2}$

37. $\lim_{x\to 7^-} \dfrac{7-x}{|x-7|}$

38. $\lim_{x\to 0^-} \dfrac{x}{x-|x|}$

39. $\lim_{x\to 2^+} \dfrac{2-x}{|x-2|}$

40. Let $G(u)=u^2+u$. Compute $\lim_{u\to 2} \dfrac{u^2-2u}{G(u-3)}$

41. Let $h(y)=y^2-3$. Compute $\lim_{x\to -2} \dfrac{x+2}{h(2x)-h(x+6)}$

42. Let $g(x)=\sqrt{x}$. Compute $\lim_{s\to 1} \dfrac{g(s^2+8)-3}{s-1}$

43. Let $f(t)=\dfrac{1}{t}$. Compute $\lim_{t\to 2} \dfrac{f(t-1)-2f(t)}{t^2-4}$

Derivatives: Use the **limit definition of the derivative** to compute these derivatives:

44. $f(x)=-4x-x^2-3$ Find $f'(x)$

45. $g(x)=\dfrac{-3}{x}$ Find $g'(x)$

46. $R(x)=x^3$ Find $R'(x)$

47. $G(x)=\dfrac{1}{x^2}$ Find $G'(x)$

48. $f(x)=\sqrt{x-7}$ Find $f'(x)$

49. $g(x)=\sqrt{7-3x}$ Find $g'(x)$

Tangent Lines: Use the limit definition of the derivative when computing derivatives here:

50. Find an equation for the tangent line to the graph of $f(x)=\dfrac{1}{x-1}$ at the point $(0,-1)$.

51. Find an equation for the tangent line to the graph of $g(x)=\dfrac{1}{x+1}$ at the point $\left(1,\dfrac{1}{2}\right)$.

52. Find an equation for the tangent line to the graph of $y=\dfrac{3}{x}+1$ when $x=1$.

Piecewise-defined functions Answer the questions (and **justify** your answers) about each of the following piecewise defined functions.

53. Let $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 3-x & \text{if } x > 1 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$$\lim_{x\to 2} f(x) = \qquad \lim_{x\to 1} f(x) = \qquad \lim_{x\to 0} f(x) =$$

54. Let $g(x) = \begin{cases} \dfrac{1}{x-4} & \text{if } x < 2 \\ \dfrac{1}{x} & \text{if } x \ge 2 \end{cases}$

Sketch the graph. Find the numbers at which g is discontinuous. Evaluate:

$$\lim_{x\to 1} g(x) = \qquad \lim_{x\to 2} g(x) =$$

55. Let $f(t) = \begin{cases} t-3 & \text{if } t \le 3 \\ 3-t & \text{if } 3 < t < 5 \\ 1 & \text{if } t = 5 \\ 3-t & \text{if } t > 5 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$$\lim_{t\to 3} f(t) = \qquad \lim_{t\to 0} f(t) = \qquad \lim_{t\to 5} f(t) =$$

56. Let $H(x) = \begin{cases} x-1 & \text{if } x < 2 \\ 1 & \text{if } 2 < x < 4 \\ 3 & \text{if } x = 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$

Sketch the graph. Find the numbers at which H is discontinuous. Evaluate:

$$\lim_{x\to 0} H(x) = \qquad \lim_{x\to 2} H(x) = \qquad \lim_{x\to 4} H(x) = \qquad H(4) =$$

57. Let $h(x) = \begin{cases} \dfrac{8}{x+2} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{1}{2}x - 4 & \text{if } 0 < x \le 16 \\ \sqrt{x} & \text{if } x > 16 \end{cases}$

Sketch the graph. Find the numbers at which h is discontinuous. Evaluate:

$$\lim_{x\to -2} h(x) = \qquad \lim_{x\to 0} h(x) = \qquad \lim_{x\to 16} h(x) =$$