Practice Problems for the Final Exam
(Warning: only covers material since Midterm Exam 3.)

Instructions: The point of this set of practice problems is **NOT** that you should plan to do all of these problems. Instead, the point is that you should skip around and try various different types of problems. And if you find you could use more practice with a particular type of problem, you should be able to find several more like it here.

So don't try to do all of these problems. But try to do a lot of them — a broad variety of them, but also extras on any particular topics that you find you could use the most practice on.

Properties of e^x and $\ln x$

1. Simplify each of the following expressions:

(a) $\ln(e^{\ln e})$ (b) $\ln\left|\ln\frac{1}{e}\right|$

2. Solve each of the the following equations for x:

(a) $\ln(\ln x) = 1$ (b) $\ln(x^2) = 2 + \ln x$ (c) $e^{3x-4} = 7$

3. Decide whether each statement is True or False. Explain why or why not.

(a) $(e^x)^2 = e^{x^2}$ (b) $\ln 5 - \ln 3 = \ln 2$ (c) $(\ln x)(\ln x) = \ln(x^2)$

Derivatives, tangent lines, etc.

4. Compute the derivatives of the following functions. (Hint: You may want to simplify first.)

(a) $f(x) = \ln(5xe^{-5x})$ (b) $g(x) = e^{\left(\ln(x^2+x) - \ln x\right)}$ (c) $h(x) = \ln\left(\frac{xe^x}{\sqrt{e^{7x}}}\right)$

5. Find the equation of the tangent line to the curve $y = (x+2)e^{-x}$ at the point $(0, 2)$.

6. Find the equation of the tangent line to the curve $y = \ln(xe^{-3x})$ at the point $(1, -3)$.

7. Find an equation for the line tangent to $y = 4\sqrt{\ln x}$ at the point where $x = e$.

8. Let $y = \dfrac{\ln x}{1+x^2}$, find $f'(1)$.

9. Let $f(x) = x \ln x$ with $x > 0$. Where is $f(x)$ concave up?

10. Let $x^2e^y = \ln(xy)$. Find $\dfrac{dy}{dx}$.

11. Find all critical numbers of the function $f(x) = (x^2 - 7)e^{-x}$, and classify each as local maximum, local minimum, or neither.

12. Let $f(x) = x^4 e^{-x}$. For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.
Take my word that $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = +\infty$

Integrals: Evaluate the following definite and indefinite integrals.

13. $\int (x-3)\sqrt{x^2-6x+\pi}\,dx$

14. $\int \dfrac{y^3+y-1}{y^4+2y^2-4y+3}\,dy$

15. $\int_1^2 \dfrac{(x+1)(x-1)}{x^3}\,dx$

16. $\int \sec^2(3x)\sin(\tan(3x))\,dx$

17. $\int_0^2 \dfrac{x}{\sqrt{2x^2+1}}\,dx$

18. $\int (x-1)\csc^2(x^2-2x)\,dx$

19. $\int_2^4 \dfrac{1}{x^2}\cos\left(\dfrac{\pi}{x}\right)\,dx$

20. $\int_0^{\pi/2} (\sin x+\cos x)^2\,dx$

21. $\int_1^e \dfrac{\sin(\pi\ln x)}{x}\,dx$

22. $\int e^{2x}\cos(e^{2x}+1)\,dx$

23. $\int x(x^2+1)^{14}\,dx$

24. $\int \sin(4x)\cos(4x)\,dx$

25. $\int \dfrac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

26. $\int_1^{e^3} \dfrac{1}{x}\sqrt{1+\ln x}\,dx$

27. $\int \dfrac{1}{(x+1)\ln(x+1)}\,dx$

28. $\int \dfrac{\sin x}{7+\cos x}\,dx$

29. $\int \dfrac{6e^x}{e^x+7}\,dx$

30. $\int \dfrac{e^{\ln(\sin x)}}{e^{\ln(\cos x+7)}}\,dx$

31. $\int \ln(e^{x^2}e^x e^7)\,dx$

32. $\int \dfrac{6x+3}{x^2+x-5}\,dx$

33. $\int \dfrac{1}{1-2x}\,dx$

34. $\int e^{3x+1}\,dx$

35. $\int \dfrac{e^{-1/x^7}}{x^8}\,dx$

36. $\int \dfrac{1}{e^x}\,dx$

37. $\int_0^1 \dfrac{1}{7x+1}\,dx$

38. $\int_e^{e^2} \dfrac{1}{x(\ln x)^2}\,dx$

39. $\int_{\ln 4}^{\ln 7} 9e^{2x}\,dx$

40. $\int_0^{\ln 3} \left(2+\dfrac{1}{e^x}\right)^2\,dx$

41. $\int \dfrac{we^{w^2}}{17+e^{w^2}}\,dw$

42. $\int_{\ln 2}^{\ln 3} e^{2x}\,dx$

43. $\int \dfrac{e^{-x}\ln(1+e^{-x})}{1+e^{-x}}\,dx$

44. $\int_e^{e^4} \dfrac{1}{x\sqrt{\ln x}}\,dx$

45. $\int (e^{3x}+e^{-7x})^2\,dx$