

In each test statement below, C can be any constant you like. Usually it is set to 1 or 0.

n th term divergence test (NTDT)	§11.2	<p>If $\lim_{n \rightarrow \infty} a_n \neq 0$, then $\sum_{n=C}^{\infty} a_n$ diverges.</p> <ul style="list-style-type: none"> • NTDT can show divergence, but does not ever show convergence. • You should usually try this test first. However, it will often be inconclusive.
Integral test (IT) (positive series only)	§11.3	<p>If $a_n = f(n)$, where $f(x)$ is a function that is positive and decreasing for sufficiently large x, then If $\int_C^{\infty} f(x) dx$ converges, then so does $\sum_{n=C}^{\infty} a_n$. If $\int_C^{\infty} f(x) dx$ diverges, then so does $\sum_{n=C}^{\infty} a_n$.</p> <ul style="list-style-type: none"> • IT is a good choice when you see a convenient way to integrate the function expressing the terms of the series.
Comparison Test (CT) (positive series only)	§11.4	<p>If $0 \leq a_n \leq b_n$ for all sufficiently large n, and $\sum_{n=C}^{\infty} b_n$ converges, then $\sum_{n=C}^{\infty} a_n$ converges as well. If $0 \leq b_n \leq a_n$ for all sufficiently large n, and $\sum_{n=C}^{\infty} a_n$ diverges, then $\sum_{n=C}^{\infty} b_n$ diverges as well.</p> <ul style="list-style-type: none"> • Typically the first step in applying this test is deciding what the “dominant terms” in the numerator and denominator are, and relating the other terms to them. • Be careful about which way the inequalities go!
Limit Comparison Test (LCT) (positive series only)	§11.4	<p>If a_n and b_n are all positive (for sufficiently large n), and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n}$ exists and is nonzero, then the two series $\sum_{n=C}^{\infty} a_n$ and $\sum_{n=C}^{\infty} b_n$ either both converge or both diverge.</p> <ul style="list-style-type: none"> • Typically the first step in applying LCT is identifying the series to compare to. The best way to do this is usually to identify the fastest-growing terms, and discard the rest. • The advantage of LCT over CT is that you must worry less about which way inequalities should go. The disadvantage is that it is necessary to evaluate a limit along the way, which could become technical in some situations.

Absolute Convergence Test (ACT)	§11.6	<p>If $\sum_{n=C}^{\infty} a_n$ converges, then $\sum_{n=C}^{\infty} a_n$ also converges (it is said to “converge absolutely”).</p> <ul style="list-style-type: none"> • Careful! The converse is not true. Sometimes $\sum a_n$ diverges, but $\sum a_n$ converges. In this case, $\sum a_n$ is said to “converge conditionally.” • ACT is usually used in conjunction with one of the tests that applies specifically to series with positive terms (IT, CT, or LCT). • This is often a good “first stop” for series with positive and negative terms. You’ll either find that the series converges absolutely (and be done), or find that the series of absolute values diverges, in which case it’s still possible that the series converges conditionally.
Alternating Series Test (AST)	§11.5	<p>If the terms a_n alternate in sign (usually tipped off by a factor of $(-1)^n$), and both of the following hold:</p> <ol style="list-style-type: none"> 1. $\lim_{n \rightarrow \infty} a_n = 0$ (i.e. the NTDT is inconclusive) 2. a_n is eventually decreasing <p>then the series $\sum_{n=C}^{\infty} a_n$ converges.</p> <ul style="list-style-type: none"> • Sometimes you can check condition 2 by hand; other times you may want to compute a derivative to check. • You should check condition 1 first. If it fails, then you will already be done, because NTDT will show off the bat that the series diverges.
Ratio Test (RT)	§11.6	<p>Let $L = \lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right$.</p> <ol style="list-style-type: none"> 1. If $L > 1$, then $\sum_{n=C}^{\infty} a_n$ diverges. (Note: in this case, the NTDT will also show that the series diverges). 2. If $L < 1$, then $\sum_{n=C}^{\infty} a_n$ converges absolutely. 3. If $L = 1$, or the limit does not exist, then RT is inconclusive. <ul style="list-style-type: none"> • RT is often well-suited to series where the formula for a_n is built out of factorials, exponential expressions, and polynomials. • RT will always be inconclusive for a conditionally convergent series.