> **Goal**   connect power series and functions; manipulate power series with differentiation and integration; evaluate some non-geometric sums
> **Reference:** §11.9

> **Note**   There is no assignment due Wednesday of this week; this one is due Friday. But it is longer than usual, so start early! The following problems can be done with material already covered by Friday 4/7, so you may want to start with them: 1, 2, 5, 6, 7, 9, 10, 11, 19.

## Problems to hand in

Find the Series Representation for the following functions using *substitution* and determine the Radius of Convergence $R$. Simplify.

1. $\dfrac{1}{1+x^2}$
2. $\dfrac{x^2}{x^4+16}$
3. $x^3\cos\left(x^2\right)$
4. $5x^2\sin\left(5x\right)$

5. $\dfrac{d}{dx}\left(x^3\arctan(7x)\right)$
6. $\displaystyle\int x^3\arctan(7x)\,dx$
7. $\dfrac{d}{dx}x^2\ln(1+6x)$
8. $\displaystyle\int x^4 e^{-x^3}\,dx$

9. Find the Series Representation for $f(x) = \dfrac{1}{(1+x)^2}$.

> **Hint**   $\dfrac{1}{(1+x)^2} = \dfrac{d}{dx}\left(-\dfrac{1}{1+x}\right).$

10.  Prove the Power Series Representation formula for $\arctan x$. You may follow the arugment shown in class.

11. Find Series Representation for $\ln(5-x)$. Solve for $C$ and the Radius $R$.

> **Hint**   $\ln(5-x) = \displaystyle\int \dfrac{-1}{5-x}\,dx.$

12. Find the MacLaurin Series for $f(x) = e^{-2x}$ using two different methods.
**First**, using the *Definition* of the MacLaurin Series.
**Second**, use Substitution into a known series. Your answers should be in Sigma notation.

13. You do **not** need to state the Radius. Answers should be in Sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here.

You may use the fact that $\boxed{\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$ without extra justification.

(a) Use the Definition to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Use Differentiation to compute the Series for $F(x) = \cos x$.

(c) Use Integration to compute the Series for $F(x) = \cos x$.

> **Note**   Make sure to include the $+C$, and solve for it.

Find the Sum of each of the following Series, which do converge.

14. $\displaystyle\sum_{n=0}^{\infty} \frac{7^n}{n!}$          15. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \, 5^n}{n!}$          16. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

17. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \, \pi^{2n}}{6^{2n} \, (2n)!}$          18. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n \, n!}$          19. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

20. $1 - \ln 2 + \dfrac{(\ln 2)^2}{2!} - \dfrac{(\ln 2)^3}{3!} + \ldots$          21. $3 + \dfrac{9}{2!} + \dfrac{27}{3!} + \dfrac{81}{4!} + \ldots$