

**Goal** estimate functions and definite integrals by Taylor approximations and the ASET. Also review interval and radius of convergence.

**Reference:** §11.10, 11.11

**Note** For these problems, you should use a handy theorem that we'll mention in class on Friday. Here it is, in case you are starting early (see also p. 775 of the textbook):

**Alternating Series Estimation Theorem (ASET):** If  $\sum_{n=0}^{\infty} (-1)^n a_n$  is an *alternating* series that passes the three conditions of the alternating series test ( $a_n$  positive,  $a_n$  decreasing,  $a_n \rightarrow 0$ ), then the error of partial sum  $\sum_{n=1}^m (-1)^n a_n$  is less than  $a_{n+1}$ .

The “error of the partial sum” means the difference between it and the sums. Less formally: the error of any partial sum is less than the absolute value of the next term in the series.

1. Use Series to Estimate  $\frac{1}{e}$  with error less than  $\frac{1}{20}$ . Justify.
2. Use Series to Estimate  $\frac{1}{e}$  with error less than  $\frac{1}{100}$ . Justify. (Can reuse work from 1)
3. Use Series to Estimate  $\frac{1}{e}$  with error less than  $\frac{1}{500}$ . Justify. (Can reuse work from 1)
4. Use Series to Estimate  $\sin(1)$  with error less than  $\frac{1}{1000}$ . Justify.
5. Use Series to Estimate  $e^{-\frac{1}{3}}$  with error less than  $\frac{1}{100}$ . Justify.
6. Use Series to Estimate  $\arctan\left(\frac{1}{2}\right)$  with error less than  $\frac{1}{100}$ . Justify.
7. Use Series to Estimate  $\int_0^1 x \ln(1+x^3) dx$  with error less than  $\frac{1}{20}$ . Justify.
8. Use Series to Estimate  $\int_0^1 x \sin(x^2) dx$  with error less than  $\frac{1}{1000}$ . Justify.

Review: Find the Interval and Radius of Convergence for each of the following.

9.  $\sum_{n=1}^{\infty} (n!)^2 (3x-7)^n$

10.  $\sum_{n=1}^{\infty} \frac{(-1)^n (5x-2)^n}{n^3 8^n}$

11.  $\sum_{n=1}^{\infty} \frac{(x-7)^n}{n! \sqrt{n}}$