Goal Evaluate trigonometric integrals and perform trigonometric substitution
Reference: §7.2, §7.3

Examples to study first

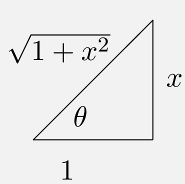
Note These examples show trigonometric substitution techniques that we will not discuss in detail until Friday 2/17, so you may want to wait until then to examine them.

Example Evaluate $\displaystyle\int \frac{1}{[1+x^2]^{\frac{7}{2}}}\,dx$

Solution We perform a "tangent substitution" (see the table and reference triangle below), followed by a substitution.

$$\begin{aligned}
\int \frac{1}{[1+x^2]^{\frac{7}{2}}}\,dx &= \int \frac{1}{(1+\tan^2\theta)^{\frac{7}{2}}}\cdot \sec^2\theta\,d\theta &&= \int \frac{1}{(\sec^2\theta)^{\frac{7}{2}}}\cdot \sec^2\theta\,d\theta \\
&= \int \frac{1}{(\sqrt{\sec^2\theta})^7}\cdot \sec^2\theta\,d\theta &&= \int \frac{1}{(\sec\theta)^7}\cdot \sec^2\theta\,d\theta \\
&= \int \frac{\sec^2\theta}{\sec^7\theta}\,d\theta &&= \int \frac{1}{\sec^5\theta}\,d\theta \\
&= \int \cos^5\theta\,d\theta &&= \int \cos^4\theta\cos\theta\,d\theta \\
&= \int (1-\sin^2\theta)^2\cos\theta\,d\theta &&= \int (1-w^2)^2\,dw \\
&= \int 1-2w^2+w^4\,dw &&= w-\frac{2w^3}{3}+\frac{w^5}{5}+C \\
&= \sin\theta-\frac{2\sin^3\theta}{3}+\frac{\sin^5\theta}{5}+C \\
&= \boxed{\frac{x}{\sqrt{1+x^2}}-\frac{2}{3}\left(\frac{x}{\sqrt{1+x^2}}\right)^3+\frac{1}{5}\left(\frac{x}{\sqrt{1+x^2}}\right)^5+C}
\end{aligned}$$

Trig. Sub $\boxed{\begin{array}{l} x=\tan\theta \\ dx=\sec^2\theta d\theta \end{array}}$



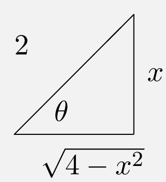
$\boxed{\begin{array}{l} w=\sin\theta \\ dw=\cos\theta\,d\theta \end{array}}$

Example Evaluate $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$.

Solution Here we use a sine substitution; see the box below.

$$\begin{aligned}
\int \frac{x^2}{\sqrt{4-x^2}}\,dx &= \int \frac{(2\sin\theta)^2}{\sqrt{4-4\sin^2\theta}}\,2\cos\theta\,d\theta &&= \int \frac{4\sin^2\theta}{\sqrt{4(1-\sin^2\theta)}}\,2\cos\theta\,d\theta \\
&= 4\int \frac{\sin^2\theta}{\sqrt{4}\sqrt{\cos^2\theta}}\,2\cos\theta\,d\theta &&= 4\int \frac{\sin^2\theta}{2\cos\theta}\,2\cos\theta\,d\theta \\
&= 4\int \sin^2\theta\,d\theta &&= 4\int \frac{1-\cos(2\theta)}{2}\,d\theta \\
&= 2\int 1-\cos(2\theta)\,d\theta &&= 2\left(\theta - \frac{\sin(2\theta)}{2}\right)+C \\
&= 2\left(\theta - \frac{2\sin\theta\cos\theta}{2}\right)+C &&= 2\left(\theta - \sin\theta\cos\theta\right)+C \\
&= \boxed{2\left[\arcsin\left(\frac{x}{2}\right) - \left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right)\right]+C}
\end{aligned}$$

Trig. Substitute $\boxed{\begin{aligned} x &= 2\sin\theta \\ dx &= 2\cos\theta d\theta \end{aligned}}$



Problems to hand in

Compute each of the following Integrals. Simplify.

1. $\int \sin^2 x\ \cos^3 x\ dx$
2. $\int_0^{\frac{\pi}{2}} \sin^5 x\ dx$
3. $\int_0^{\frac{\pi}{2}} \cos^2\theta\ d\theta$
4. $\int_0^{\frac{\pi}{2}} \sin^2 x\ \cos^2 x\ dx$
5. $\int x\ \sin^2 x\ dx$
6. $\int_0^1 x^3\sqrt{1-x^2}\ dx$ use Trig Sub
7. $\int \sqrt{9-x^2}\ dx$
8. $\int \frac{1}{(4+x^2)^{\frac{5}{2}}}\ dx$
9. $\int x\ \arcsin x\ dx$