Topics trig. subs. involving secant and tangent; completing the square; partial fractions
Reference: §7.3, §7.4

Examples to study first

Example Evaluate $\int \frac{6}{x^2+2x+8}\,dx$.

Solution

$$\int \frac{6}{x^2+2x+8}\,dx \overset{\substack{\text{complete}\\ \text{square}}}{=} \int \frac{6}{(x+1)^2+7}\,dx \qquad \boxed{\begin{aligned} u &= x+1 \\ du &= dx \end{aligned}}$$

$$= 6\int \frac{1}{u^2+7}\,du$$

$$= 6\left(\frac{1}{\sqrt{7}}\right)\arctan\left(\frac{u}{\sqrt{7}}\right)+C$$

(“a-rule”, or by substituting $v = u/\sqrt{7}$)

$$= \boxed{\frac{6}{\sqrt{7}}\arctan\left(\frac{x+1}{\sqrt{7}}\right)+C}$$

Example Evaluate $\int_{-3}^{1} \frac{6}{x^2+2x-8}\,dx$.

Solution

$$\int_{-3}^{1} \frac{6}{x^2+2x-8}\,dx \overset{\text{factor}}{=} \int_{-3}^{1} \frac{6}{(x-2)(x+4)}\,dx$$

$$\overset{\text{PFD}}{=} \int_{-3}^{1} \frac{1}{x-2} - \frac{1}{x+4}\,dx \qquad \text{(see algebra below)}$$

$$= \ln|x-2| - \ln|x+4|\Big|_{-3}^{1}$$

$$= \ln|-1| - \ln 5 - (\ln|-5| - \ln 1)$$

$$= 0 - \ln 5 - \ln 5 + 0$$

$$= \boxed{-2\ln 5}$$

Partial Fractions Decomposition:
$\frac{6}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$
Clearing the denominator yields:
$6 = A(x+4) + B(x-2) = Ax + 4A + Bx - 2B = (A+B)x + (4A-2B)$
so that $A+B=0$ and $4A-2B=6$.
Solve to obtain $A=1$ and $B=-1$.

Problems to hand in

Compute each of the following Integrals. Simplify when possible.

1. $\displaystyle\int \frac{1}{\sqrt{4-4x-x^2}}\,dx$

2. $\displaystyle\int_{-1}^{1} \frac{1}{x^2+4x+7}\,dx$

3. $\displaystyle\int \sqrt{3-2x-x^2}\,dx$

4. $\displaystyle\int \frac{x+4}{x^2+2x+5}\,dx$

5. $\displaystyle\int_3^5 \frac{6}{x^2-4x+7}\,dx$

6. $\displaystyle\int_0^3 \frac{6}{x^2-4x-5}\,dx$

7. $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

8. $\displaystyle\int \frac{\arctan x}{x^2}\,dx = \int \arctan x \cdot \left(x^{-2}\right)\,dx$

9. $\displaystyle\int_{\ln 2}^{\ln 5} \frac{2e^x}{e^{2x}-1}\,dx$

10. $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\,dx$

↑ hint: $$\frac{2}{u^2-1} = \frac{2}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}$$

↑ use PFD $$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$