Problem Set 1 Math 19, Fall 2014

Each problem is labeled with the date of the lecture after which you should attempt the problem (or "review" if it reviews calculus 1 material). Of course, you are encouraged to begin early.

Please work together and come to office hours to ask questions! Remember that when you finish working, you must write your solutions entirely by yourself.

Also feel free to visit the Math *Math Resource Center*, which is in Foxboro Auditorium from 8 to 10pm each night from Monday to Thursday. There will be tutors to help you, and other students to collaborate with.

- 1. What is the average value of $\sin \vartheta$, for ϑ in the interval $[0, \pi]$? (review)
- 2. An airplane is cruising at a constant elevation of 5 kilometers. An observer on the ground is standing under the flight path of the plane (that is, the plane will fly over the observer's head). The observer uses a radar device to determine that the plane is 13 kilometers away from her, and that its distance to her is decreasing at a rate of 200 meters per second. How fast is the airplane traveling through the air? (review)
- 3. Evaluate the following integrals.

(a)
$$\int_0^{\ln 2} \frac{e^{3x}}{1 + e^{6x}} dx$$

 (b) $\int_2^3 \frac{dx}{x \ln x}$
 (c) $\int_0^1 x (1 - x)^{2/3} dx$
 (d) $\int x e^{-x^2/2} dx$

(review)

4. A firm sells widgets. It costs the firm 5 dollars to produce a widget. The firm determines that if they sell widgets for p dollars per widget, then they will sell approximately 2^{-p} million widgets per year. Which price p will generate the most profit per year for the firm? (review)

5. Evaluate
$$\lim_{\vartheta \to \pi/2} (\sin \vartheta)^{\tan^2 \vartheta}$$
. (review)

6. Evaluate the following integrals.

(a)
$$\int_0^{\pi} x \sin x \, dx$$

(b) $\int_{-\pi/2}^{\pi/2} x^2 \cos x \, dx$
(c) $\int x^2 \, 3^x \, dx$
(d) $\int_1^{10} x \ln x \, dx$

(W 9/3)

7. Evaluate
$$\int 5^x \cdot \sin(x/5) dx$$
. (W 9/3)

8. Evaluate
$$\int_0^{\pi^2} \sin(\sqrt{x}) dx$$
. (W 9/3)

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9. Compute the following integrals.

(a)
$$\int_0^{\pi/2} \sin x \, \cos^5 x \, dx$$

$$(d) \int_{-\pi/6}^{\pi/6} \cos^3 x \ dx$$

(b)
$$\int \sqrt{\cos x} \sin x \, dx$$

(e)
$$\int_{-\pi/3}^{\pi/3} \sec^4 x \ dx$$

(c)
$$\int_0^{4\pi} \sin^2 x \cos^2 x \, dx$$

(f)
$$\int \tan^4 x \ dx$$

(F 9/5)

10. Suppose that $f(x) = A + B \sin x + C \sin(2x)$, where A, B, C are constants. Compute the following integrals (express your answer in terms of A, B, and C).

(a)
$$\int_{0}^{2\pi} f(x) \ dx$$

(d)
$$\int_0^{2\pi} f(x) \sin(2x) \ dx$$

(b)
$$\int_0^{2\pi} f(x) \sin x \, dx$$

(e)
$$\int_0^{2\pi} f(x) \cos(2x) \ dx$$

(c)
$$\int_0^{2\pi} f(x) \cos x \ dx$$

(f)
$$\int_{0}^{2\pi} f(x)^2 dx$$

(F 9/5)

Hint: we mentioned a fact in class that substantially reduces the labor in computing these integrals.

11. Evaluate the following integrals.

(a)
$$\int_{-1}^{1} \frac{dx}{\sqrt{9-x^2}}$$

(c)
$$\int_{-1}^{1} \frac{dx}{(1+x^2)^2}$$

(b)
$$\int \frac{dx}{\sqrt{x^2 - 3}}$$

(d)
$$\int_{\sqrt{2}}^{2} \frac{(x^2 - 2)^{3/2}}{x} dx$$

(M 9/8)

12. Evaluate
$$\int \frac{dx}{x^2 - 2x + 2}.$$

(M 9/8)