Please staple your assignment!

1. For each point expressed in polar coordinates, find a second way to express the same point in polar coordinates.

   (a) $(r, \theta) = (-2, \pi/27)$

   (b) $(r, \theta) = (2, 3\pi)$

   (c) $(r, \theta) = (4, -\pi/9)$

   (d) $(r, \theta) = (5, \frac{8}{7}\pi)$

2. Consider the polar curve described by $r = 2\sqrt{\cos(2\theta)}$, for $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$. For some parts of this problem, you may want to use the identity $\cos(2\theta) = \cos^2\theta - \sin^2\theta$.

   (a) Find an equation for this curve in rectangular coordinates (x and y).

   (b) Find the area enclosed by this curve.

   (c) Write an integral expressing the length of this curve. Evaluate this integral using Wolfram Alpha or another computer system (do not attempt to evaluate it by hand).

3. Let A and B be constants. Convert the equation $(x-A)^2+(y-B)^2 = A^2+B^2$ to an equation of the form $r = F(\theta)$.

4. Sketch the curves $r = \sin(n\theta)$ for $n = 1, 2, 3, 4, 5, 6$ (use a computer or graphing calculator). Determine the pattern in the number of "lobes" of the resulting curve. Compute the area of one "lobe," in terms of n.

5. Find a polar equation (in the form $r = F(\theta)$) for a hyperbola whose asymptotes are parallel to the rays $\theta = \pm\frac{\pi}{4}$. Convert this equation to rectangular coordinates (x and y).

6. Let A be a constant. What are the possible shapes of the curve described by $r = \frac{1}{A-\cos\theta}$? For which values of A does each shape occur?

7. For each equation, find all complex solutions. Express your answers in both rectangular form ($x + iy$) and polar form ($re^{i\theta}$).

   (a) $z^8 = 1$

   (b) $z^3 = 8i$

8. Let z be a complex number, and $\bar{z}$ its complex conjugate. Determine all possible values of each of the following.

   (a) $z + \bar{z}$

   (b) $z - \bar{z}$

   (c) $z \cdot \bar{z}$

   (d) $z/\bar{z}$

9. Find a pair of complex numbers w and z that satisfy the following equations.

$$\begin{aligned}(1+i)z + (2+i)w &= 4+3i\\ (1-i)z + (5+i)w &= 0\end{aligned}$$

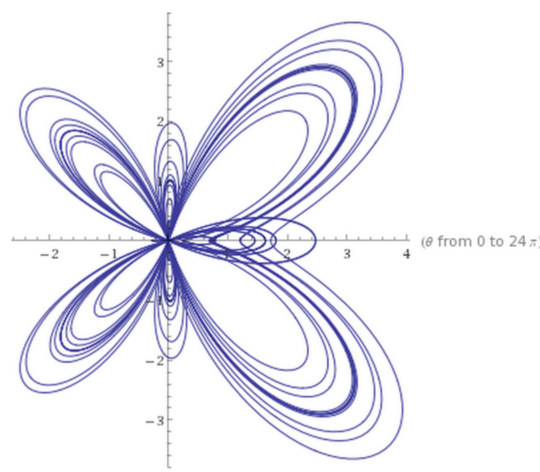
10. For each of the four functions in the left column, identify the equation in the right column which is true about this function (there is exactly one equation for each function).

A. $f(x) = e^{2x}$	I. $f''(x) - 2f'(x) + f(x) = 0$
B. $f(x) = x \cdot e^x$	II. $f''(x) + 2f'(x) + 2f(x) = 0$
C. $f(x) = e^{-x} \cos x$	III. $f''(x) - f'(x) - 2f(x) = 0$
D. $f(x) = e^x \sin x$	IV. $f''(x) - 2f'(x) + 2f(x) = 0$

Note. These equations are called *differential equations*, and we will begin to discuss them in class on Friday the third. We will consider them in more detail after the midterm.

11. The following polar equation is called *Dr. Fay's Butterfly,* according to the textbook. The picture is generated with Wolfram Alpha.

$$r = e^{\cos\theta} - 2\cos(4\theta) + \sin^5\left(\frac{\theta}{12}\right)$$



(a) Try modifying the "12" in this equation to other values (e.g. change it to 1, 2, or values larger than 12). How does this affect the appearance of the curve? (Use Wolfram Alpha or a computer program or calculator of your choice)

(b) Try modifying the number "2" in this equation (e.g. change it to 1.9, 1, or values larger than 2). How does this affect the appearance of the curve?

(c) Modify the equation in some other way of your choice, and describe how the butterfly changes in response.

This problem will be graded very generously, and a few words are sufficient for your answers. You are just meant to experiment a little and see what happens.