The first five problems cover material discussed before the midterm. The remaining three problems concern material to be discussed on Wednesday 10/15.

1. For each of the following differential equations, determine the characteristic equation. You do not need to solve the characteristic equation.

   (a) $f'(x) - 27f(x) = 0$
   (b) $f'''(x) + f''(x) + f(x) = 0$
   (c) $f^{(7)}(x) + f'(x) = 0$
   (d) $f''(x) - 4f'(x) + 16f(x) = 0$

2. Find a nonzero real solution to each of the following (linear, homogeneous) differential equations.

   (a) $f''(x) + 8f'(x) + 7f(x) = 0$
   (b) $f''(x) + 8f'(x) + 16f(x) = 0$
   (c) $f''(x) + 8f'(x) + 20f(x) = 0$
   (d) $f''(x) + 8f'(x) + 116f(x) = 0$

3. Find a nonzero real solution to the differential equation $f'''(x) = -f(x)$ that is not a constant multiple of e^{-x}.

4. Consider the function $f(x) = e^{-7x}\cos(2x)$. Find a second order linear homogeneous differential equation that is satisfied by $f(x)$.

5. Consider the differential equation $y''(t) + dy'(t) + ky(t) = 0$. We mentioned in class that this equation describes the motion of a damped spring. A damped spring is called *overdamped* if the characteristic equation of this differential equation has only real solutions.

   (a) Suppose that $k = 16$. How large must the constant d be in order for the spring to be over damped?
   (b) Suppose that $d = 6$. For which values of k will the spring be overdamped?

6. Find the general solution of each of the following first order differential equations. Note that not all of these are linear and homogeneous.

   (a) $f'(x) + 5f(x) = 0$
   (b) $f'(x) = 5\sin x$
   (c) $f'(x) = 3f(x)$
   (d) $f'(x) = 3x^2$

7. Find the general solution to each of the four differential equations in problem 2.

8. For each of the four differential equations in problem 2, find the unique solution $f(x)$ that satisfies the following initial conditions.

$$f(0) = 0$$

$$f'(0) = 6$$