1. Use the **comparison test** to determine whether or not each of the following infinite sums or improper integrals converge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+4}$

   (b) $\displaystyle\int_{2}^{\infty} \frac{e^{-x}}{x-1}\, dx$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{1}{n}\right)$

   (d) $\displaystyle\int_{1}^{\infty} x^{-x}\, dx$

2. Use the **ratio test** to determine whether or not each of the following series converge.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{11^n}{n!}$

   (b) $\displaystyle\sum_{m=0}^{\infty} (-1)^m \frac{m!}{1000^m}$

   (c) $\displaystyle\sum_{n=5}^{\infty} \frac{5^n}{n^{100}}$

   (d) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{k!}{k^k}$

3. For each of the following alternating series, determine whether it is divergent, conditionally convergent, or absolutely convergent.

   (a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n e^{-n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

   (c) $\displaystyle\sum_{\ell=0}^{\infty} (-1)^\ell \cdot \frac{\ell+1}{2\ell+1}$

   (d) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{1}{\sqrt{k^2-2}}$

4. Determine whether or not each of the following series converge, using a method of your choice.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n}+10^{27}}$

   (c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{1}{3n+1}$

   (d) $\displaystyle\sum_{m=1}^{\infty} \frac{1}{5m+3}$

   (e) $\displaystyle\sum_{\ell=1}^{\infty} \frac{1}{\ell^3}$

   (f) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{10^k}{(2k)!}$

5. A radioactive particle will decay randomly after some number of days. The probability that it will decay on any particular day (given that it has not yet decayed) is 0.001. From this it follows that the probability that the particle will decay on day n is

$$p_n = 0.999^{n-1} \cdot 0.001.$$

   (a) Compute $\displaystyle\sum_{n=1}^{\infty} p_n$.

(b) Compute the *expected value* of the day on which the particle decays. This is given by the formula

$$\mu = \sum_{n=1}^{\infty} (n \cdot p_n).$$

The expected value tells, roughly speaking, how long you should expect this particle to last before it decays. In other words, given a large number of identical particles, this will be very close to the average day of decay.