

Math 19 Review Problems

12/10/14

① Find the Taylor series of  $\frac{1}{1+z^2}$  with center  $x=0$ .  
What is its radius of convergence?

② Find the quadratic approximation of  $\sqrt{1+\sin x}$  around  $x=0$ .

③ Find a series (of rational numbers) whose sum converges to

$$\int_0^2 \sin(x^2) dx$$

④ Find the Taylor series of  $f(x) = \frac{1}{2}(e^x + e^{-x})$ .

⑤ Find the Fourier series ( $2\pi$ -periodic) of  $f(x)$ , where:

$$f(x) = \begin{cases} 1 & -\pi/2 \leq x < \pi/2 \\ 0 & -\pi \leq x < -\pi/2 \text{ or } \pi/2 \leq x < \pi \end{cases}$$

$$f(x+2\pi) = f(x).$$

⑥ Find the real & complex Fourier coeffs. of  $\sin^2 x$ .  
(hint. find a way to avoid taking any integrals).

⑦ Find the steady-state ( $2\pi$ -periodic) solution to

$$f''(t) + 10f(t) = \cos t + \cos(3t)$$