1. Recall that if S is a set, the *powerset* $\mathcal{P}(S)$ of S is the set of all subsets of S.
   (a) [3 points] List the elements of $\mathcal{P}(\{1,2,3\})$.
   (b) [3 points] List the elements of $\mathcal{P}(\emptyset)$.
   (c) [3 points] List the elements of $\mathcal{P}(\mathcal{P}(\emptyset))$.
   (d) [3 points] List the elements of $\left\{S \in \mathcal{P}(\{1,2,3\}) \;\middle|\; |S| = 2\right\}$.

2. [12 points] A drawer contains 8 gray socks, 10 black socks, and 6 white socks. Suppose that you draw 10 socks from the drawer without looking at them. Prove that you will definitely draw some four socks of the same color.

3. [12 points] For each of the following statements, write the negation of the statement **using logical symbols without using the $\sim$ symbol**, and determine whether the negation is true or false. No explanations are necessary, but an explanation may earn partial credit if the answer is incorrect.
   (a) $\exists n \in \mathbb{Z}$ such that $\big(2 \nmid n\big) \wedge \big(2 \nmid (n-1)\big)$ The negation is (in symbols): ______________________
   The negation is: ☐ True ☐ False
   (b) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $y^2 \neq x$. The negation is (in symbols): ______________________
   The negation is: ☐ True ☐ False
   (c) $\forall a \in \mathbb{Z}\, \forall b \in \mathbb{Z}\, \big((a \mid b^2) \Rightarrow (a \mid b)\big)$. The negation is (in symbols): ______________________
   The negation is: ☐ True ☐ False

4. [12 points] Prove the following formula, by induction on n.

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)} = 1 - \frac{1}{n+1}.$$

5. [12 points] Let n be an integer. Prove that n is odd *if and only if* $n^2 - 1$ is divisible by 4.