The proof portfolio for this course is meant to give your practice producing highly polished mathematical communication, and also to allow you to reflect on the skills and topics that you have learned in the course. **The complete portfolio is due by the end of finals week**, on Friday May 15.

The portfolio should be submitted as a single PDF file. A A LaTeX template is provided on the course website that you may use if you wish.

The portfolio consists of two parts:

- Four carefully written and edited proofs, giving a representative sample of your best work from the course. These must be typed in LaTeX and include some introductory remarks explaining the context and stating and technical definitions or preliminary results (e.g. theorems discussed in class) that you need. The template provided for Entry 1 gives an example of what this expository material could look like.

- A roughly two page essay addressing what you have learned in the course and the role that proofs play in mathematics. When writing this essay, you might consider reflecting on how this course is different or similar to math courses you have taken before, any ways that you now think differently about your prior courses and mathematics as a whole, and your hopes and plans for learning more mathematics in the future. Do not stress about saying anything deep or clever here; you will receive full points for this part as long as you show clear effort and seem to have reflected on the course.

Choice of topics: Entry 1 of the portfolio is on an assigned topic (irrationality of $\sqrt[k]{p}$) and follows a provided template. For the other three entries, you should choose three proofs that you have written in the course that you believe display your best work and display a range of techniques and topics. You should choose proofs that are substantial enough to show your skills (i.e. don't choose proofs that can be done in a sentence or two), and try to choose examples showing a couple different proof techniques.

Revising the first entry: You have already written a draft of the first entry of the portfolio (the one you turned in with Problem Set 6). I will score and return this entry as if it were an entry in the final portfolio. My goal with this first entry is to give you feedback on how to revise your writing to meet the standards of the final portfolio. You may keep the assigned score if you wish, or you may revise this entry and submit the new version with the final portfolio (in which case only the final score will count).

Submitting and revising the second entry: I had initially planned to require student to submit a second entry during the semester as well. To give you more flexibility during the current situation, this is not longer required. However, you may submit a draft and received feedback on it if you wish. If you want to submit a draft of entry 2, you should submit it by **Friday, May 1.** I will score it and return it with comments before the beginning of finals week, and you will have the option to revise and resubmit it if you wish (in which case only the final score will count).

Intended audience: You should write the proofs as if you are writing for a student who has taken Math 220 and is familiar with the basic mechanics and techniques of mathematical argument, but may not know all the details of the topics we covered. Any technical definitions or notation should be briefly defined, and results proved in class that you need in your proof should be clearly stated

(but need not be proved in full). These should be written like you would write any essay or piece of technical writing. For example, you should write in complete sentences with correct grammar and punctuation and use technical notation when it helps you communicate clearly.

Scoring: Each entry in the proof portfolio will be scored according to the following 16-point rubric. The reflection essay will be worth 16 points as well, so the portfolio as a whole will be scored out of 80 points.

This rubric is based on a rubric written by Amalia Culiuc for use in Math 220.

Understanding of the statement (2 points)

- 0: The proof writer does not understand the statement. They are either attempting to prove the wrong fact or using wrong definitions of the objects at hand.

- 1: It is unclear that the writer fully understands the statement. Either the statement is rephrased in a modified version (perhaps including extra assumptions) or the argument indicates a misunderstanding.

- 2: The proof writer fully understands the statement that they are attempting to prove. If the statement is rephrased or rewritten in symbols, the rewriting is correct and complete. If the statement is not rewritten, it is obvious from the content of the argument that the writer is proving the right fact.

Logical soundness (5 points)

- 0: Mostly incorrect statements/implications. No sense of how to prove the result or proving a completely unrelated result.

- 1: Unconnected, but mostly true statements. Facts listed without a link to the proof. Confusing a proof with an example or providing only an intuitive explanation instead of a mathematical argument.

- 2: Mostly true statements, mostly linked to the proof and to each other, but the proof is incomplete or the argument only holds for a weaker setting (for example, "if and only if" statements missing one of the implications, proving that a statement holds for integers when asked to show it holds for all real numbers, including extra assumptions).

- 3: True statements, mostly linked to each other and to the argument, but the proof depends upon a major unjustified leap.

- 4: True statements, all related to each other and to the argument, but some may be improperly or insufficiently justified. The errors are minor and could be fixed upon a closer reading.

- 5: A correct, complete, and polished argument. All statements are true and relevant and the conclusion follows naturally from the chain of implications.

Presentation of proof (4 points)

- 0: The proof is impossible to understand, extremely informal, or illegible, variables are not defined, statements lack organization, and it is impossible to follow the writer's train of thought.

- 1: The proof is very difficult to follow. The grammar and spelling mistakes are distracting OR the proof is almost entirely symbols OR the language is too informal or imprecise to the point where the correctness of the argument cannot be assessed. Variables are not defined and their meaning is difficult to establish. A reader needs to put in considerable effort to understand what the writer is assuming and what they are proving.

- 2: The proof is not overly difficult to follow, but requires substantial proof-reading and/or revision due to grammar/spelling errors OR overuse/misuse of symbols OR misleading use of words like "thus" or "therefore" OR improper references to prior results.

- 3: The flow is sometimes unnatural or difficult to follow OR there are some grammar/spelling errors OR the sentences are incomplete and often replaced by too many symbols. Some variables may not be defined, but their meaning can be guessed from the context. Words like "thus" or "therefore" are sometimes misused. Prior results are not referenced properly.

- 4: Good flow, full and precise sentences, correct grammar and punctuation, all variables are defined when being introduced, symbols are used in moderation and it is easy to read through the argument. There is no ambiguity about what is claimed to follow from what or where the assumptions of the theorem are employed. If another theorem is employed, it is mentioned explicitly and referenced.

Exposition of context (3 points)

- 0: No exposition is provided, or what is written provides no context for the theorem and its proof.

- 1: Some context is provided for the theorem and its proof, but it is poorly explained OR it is not clear to the reader how this theorem and its proof fit into a broader context OR important terms or necessary prior results are not explained.

- 2: The context of the theorem and its proof are generally well-described, including explanations of relevant definitions and prior results. Some aspects of the exposition are unclear or misleading OR some important aspects have been left out.

- 3: Introductory comments explain the context of the theorem being proved. Relevant definitions and prior results are clearly explained, either in the proof itself or in introductory remarks. The reader can tell how this theorem (and its proof) fit into the broader context of the course and the subject.

Choice of topic (2 points)

- 0: Topic is not related to our Math 220 course.

- 1: Topic chosen is not substantial enough to adequately demonstrate writing skills learned in the course OR the techniques displayed in this proof significantly overlap with those of another entry in the portfolio.

- 2: Topic is substantial, nicely illustrates techniques and skills learned in the course, and complements the topics chosen for the other portfolio entries.