Suggested reading for this week (from the textbook): §1.1, §1.2, §1.3

Study items for PSet 1:

- Proof by contradiction that $\sqrt{2} \notin \mathbb{Q}$.
- Related irrationality proofs, e.g. $\sqrt{2} + 1 \notin \mathbb{Q}$.
- Sets and related notation, including $\in, \subseteq$ and $\emptyset$.
- Set-builder notation.
- How to work with sets of sets, e.g. indexed collections.
- Set operations: $\cap, \cup, \setminus, ^c$, and $\times$.
- Visualizing set operations using Venn diagrams.
- Unions and intersections of indexed collections; the notation $\bigcup_{i \in I} S_i$ and $\bigcap_{i \in I} S_i$.
- Statement and proof of de Morgan's laws for sets.
- Be aware that set complement notation is only defined when a "universe" U has been chosen.
- Definition and properties of the "power set" $\mathcal{P}(S)$.

Problems from the book:

- Section 1.1: 1, 3, 9

  (in 9(b), it is sufficient to answer "true" or "false;" no explanation is needed)
- Section 1.2: 1

Supplemental problems:

1. Prove that $2\sqrt{2}$ is irrational. You may assume the fact that $\sqrt{2}$ is irrational (proved in class).

2. The following paragraph claims to prove that $\sqrt{4}$ is irrational, following the same strategy that we used in class for $\sqrt{2}$ and $\sqrt{3}$. This cannot be correct, since $\sqrt{4} = 2$. Identify the specific step in the argument where the error occurs, and explain why that step is not valid.

   Suppose, for contradiction, that $\sqrt{4}$ is rational. Then by expressing it as a reduced fraction, we may write $\sqrt{4} = \frac{a}{b}$ for some integers a and b with no common factors. It follows that $4b^2 = a^2$. The left side of this equation, $4b^2$, is divisible by 4. Therefore the right side, a^2, is also divisible by 4. It follows that a is divisible by 4. Thus $a = 4c$ for some integer c, and so $4b^2 = (4c)^2 = 16c^2$. Cancelling a factor of 4, we obtain $b^2 = 4c^2$. Therefore b^2 is divisible by 4, and by the same logic as above, b itself is divisible by 4. But this shows that both a and b are divisible by 4, which is a contradiction. Therefore $\sqrt{4}$ is not rational.

3. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. For each set expressed in set-builder notation below, list all the elements of the set (write the elements within curly braces).

(a) $S = \{x \in U \mid x^2 < 46\}$

(b) $S = \{x \in U \mid x^2 - 3x = -2\}$

4. Let $V = \{-3, -2, -1, 0, 1, 2, 3\}$. Which of the following sets are equal?

(a) $A = \{n \in V \mid |n| < 2\}$

(b) $B = \{n \in V \mid n^2 \leq 1\}$

(c) $C = \{n \in V \mid n^3 = n\}$

(d) $D = \{-1, 0, 1\}$

(e) $E = \{n \in V \mid n^2 \leq n\}$

5. Define two sets as follows.

$$\begin{aligned} A &= \{1\} \\ B &= \Big\{\{1\},\ \{\{1\}\},\ \Big\{\{1\}, \{\{1\}\}\Big\}\Big\} \end{aligned}$$

For each of the following statements, determine where it is true or false. Briefly explain your answer.

(a) $A \in B$

(b) $A \subseteq B$

(c) $\{A\} \in B$

(d) $\{A\} \subseteq B$