Note: These problems are meant for you to practice with the last week of material, but you are NOT required to write up solutions or turn anything in. I will post instead solutions on the website so that you can check your work.

Suggested reading for this week (from the textbook): §8.3, §8.4

Problems from the book: (First two numbers refer to the section number)

- 8.3.2 $\left(\lim_{n\to\infty} \frac{2n-3}{4-5n}\right)$
- 8.3.3 $\left(\lim_{n\to\infty} \frac{1-n^2}{3n^2+1}\right)$
- 8.3.6 (convergent sequences of integers are eventually constant)
- 9.3.12(a) (shifting a sequence does not change the limit)

Supplemental problems:

1. Suppose that $(a_n)_{n=1}^{\infty}$ is a convergent sequence of positive real numbers, with limit L. Prove that $(\sqrt{a_n})_{n=1}^{\infty}$ is also convergent, and has limit $\sqrt{L}$.
2. Prove the "decreasing" version of the monotone convergence theorem: if $(a_n)_{n=1}^{\infty}$ is a decreasing sequence of real numbers, then $(a_n)_{n=1}^{\infty}$ converges if and only if it is bounded below.
3. (The comparison test for infinite series) Suppose that $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ are two sequences of *positive* real numbers, such that $a_n \leq b_n$ for all $n \in \mathbb{N}$. Prove that if the series $\sum_{n=1}^{\infty}$ converges, then the series $\sum_{n=1}^{\infty}$ also converges. (*Hint:* use the monotone convergence theorem.)