

Suggested reading for this week (from the textbook):

§1.4 (logic), §1.5 (quantifiers), §1.6 (implications), §2.1 (proof techniques)

Study items for PSet 2:

- Definition of a “partition.”
- Logical symbols: $\vee, \wedge, \sim, \Rightarrow, \Leftrightarrow$.
- Standard translations, e.g. “ $P \Rightarrow Q$ ” = “ $\sim P \vee Q$ ” and “ $P \Leftrightarrow Q$ ” = “ $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ ”.
- Quantifiers \forall, \exists and their meaning.
- Be able to *fluently* translate mathematical statements from English to logic notation and vice versa.
- Negating students; syntax for “moving a negation through” quantifiers and logical symbols. Make sure you understand *why* these rules hold!
- Terms: “converse” and “contrapositive.” The contrapositive is equivalent to the original statement, while the converse is not.
- Direct proofs of universal statements (“Let x be any element of S ...”)
- Direct proofs of existential statements (“Let $x =$ [something specific]...”)
- Direct proofs of implications (“Assume P ... therefore Q ”)
- Definitions of “even” and “odd” in terms of existence of an integer k .

Problems from the book:

The first two numbers refer to the section number, e.g. 1.3.3 is problem 3 in §1.3.

The summary in parentheses is just for your reference in remembering which problem is which.

- 1.2.2 (set operations on some sets of cards from a standard deck)
- 1.2.16 (sets of combination keys)
- 1.3.3 (identifying whether or not a collection is a partition)
- 1.3.5 (partitions of \mathbb{R}^2)
- 1.4.3 (using logical symbols)
- 1.5.1 (write a statement using quantifiers)
- 1.5.2 (determine truth value and negate statement with quantifiers)
- 1.6.2 (expressing implications in words)
- 1.6.3 (expressing implications in symbols)
- 1.6.6 (Converses)

Supplemental problems:

1. Write the negation of the following statement:

$$P : \forall r \in \mathbb{R}, (r > 0 \Rightarrow (\exists s \in \mathbb{R} \text{ such that } s^2 = r)).$$

2. Prove that the statement P in Supplement Problem 1 is true.
3. Prove that the sum of two even integers is an even integer (for our purposes, an even integer is defined to be an integer equal to $2k$ for some integer k).