Suggested reading for this week (from the textbook):
§4.1 through §4.4 (Combinatorics)

Study items for PSet 6:

- Know how to prove: $\gcd(a, b) = 1$ if and only if $\exists u, v \in \mathbb{Z}$ such that $au + bv = 1$.
- Know the two formulations of "Euclid's lemma" (for relatively prime integers, and for prime integers), and why one implies the other.
- The proof of the "uniqueness" half of the fundamental theorem of arithmetic.
- Some ways to use the fundamental theorem of arithmetic: divisibility in terms of prime factorization; greatest common divisors; least common multiples.
- Definition of *binomial coefficients* $\binom{n}{k}$.
- ~~Statement and proof of the binomial theorem.~~
- ~~Understand the "fundamental counting principle."~~

Problems from the book: (First two numbers refer to the section number)

- 3.2.3 ($sa + tb = 21$ and $ua + vb = 10$ imply that $\gcd(a, b) = 1$)
- 3.2.9 ($\gcd(a, b) = \gcd(a, a + b)$)
- 3.3.5 (LCM of relatively prime numbers)
- 3.3.7 (Describing squares in terms of prime factorization)
- 3.3.13 (Prove or disprove some statements related to Euclid's lemma; four parts)
- 3.3.17(a) ($\log_{10} 3 \notin \mathbb{Q}$)
- ~~4.1.4 (divisors of 2310 with exactly 3 prime factors)~~
- 4.1.5 **You may leave your answer in the form $\binom{n}{k}$.** (Ways to press two, three, or four keys on a 12-key piano)
- ~~4.2.2 (finger positions on a three-valve trumpet)~~
- ~~4.2.5 **You may leave your answer unsimplified in terms of binomial coefficients** (choices of a committee)~~

Supplemental problems:

1. Let $a, b, u, v \in \mathbb{Z}$, and suppose that $au + bv = \gcd(a, b)$. Prove that $\gcd(u, v) = 1$. This shows that the numbers u, v arising from the Euclidean algorithm are always relatively prime (even if the input numbers a, b are not).
2. Prove that if a, b, c are integers such that $\gcd(a, b) = 1$ and $ab = c^2$, then both a and b are squares. (Hint: use the fundamental theorem of arithmetic.)

Proof Portfolio, Entry 1:

In addition to the problems above, a draft of the first entry in your proof portfolio is also due on the same day, as a separate submission on Gradescope. The main purpose of this first entry is to learn how to type mathematics in LaTeX; you will be reproducing the proof of a theorem that we have covered in class. You should follow these steps. **Please do steps** 1 **and** 2 **before Wednesday's class, so that you will be ready for a LaTeX workshop that day.**

1. Either create an account on `overleaf.com` (if you wish to work from a web browser), or install a LaTeX program on your computer (if you want to work directly on your computer). You can find resources and links at Allison Tanguay's webpage, below.

   `https://www.amherst.edu/people/facstaff/atanguay/latex`

   Please visit the Q Center and speak with Allison Tanguay or Allison Randie-Cofie for help setting up LaTeX on your computer.

2. Download the file "Introduction to LaTeX (TeX file)" from Allison Tanguay's webpage (above) and paste it into your LaTeX editor. Compile it to produce a nice pdf file.

3. Try working through the "Brisk Introduction to LaTeX" linked from Allison Tanguay's webpage, and experiment with all the commands described. Or wait until the workshop on Wednesday, when the Q Center staff will demonstrate some basic techniques.

4. When you have the hang of LaTeX editing and compiling, download the file "Proof Portfolio Entry 1 template" from the course webpage. Read the instructions in blue. You will need to reproduce all the formatting and black text, and fill in the proofs yourself.

5. When you have a polished product, download the pdf (if you are using Overleaf), and then upload the pdf to Gradescope under the "Proof Portfolio Entry 1" assignment.

There is an initial learning curve with LaTeX, when it will take much more effort than handwriting would take. You will find that your efficiency will quickly improve after that, and it will be very convenient to be able to quickly and efficiently type beautiful mathematical documents.

Some advice: **a search engines is your friend!** No one, and I mean *no one*, has all the fiddly details of LaTeX memorized. If you're not sure how to do something, search for it. You will almost certainly find some examples and sample code.