Work on the problems below with one or two students nearby.

Call me over if you have questions or want to check answers!

Don't worry if some of this seems unfamiliar. The worksheet is an exercise to help you learn the material and think about **new** things. It is not a test, and you don't need to be able to do all of it right away.

1. Evaluate each of the following sums, and express your answer as a reduced fraction.

(a) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}$ $= \frac{1}{2}+\frac{1}{6} = \frac{3+1}{6} = \frac{4}{6}$

$= \boxed{\frac{2}{3}}$

(b) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}$ $\frac{1}{2}+\frac{1}{6}+\frac{1}{12} = \frac{6+2+1}{12} = \frac{9}{12}$

$= \boxed{\frac{3}{4}}$

(c) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}$ $= \frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}$

$= \frac{3}{4}+\frac{1}{20}$ (using answer in (b))

$= \frac{15+1}{20} = \frac{16}{20} = \boxed{\frac{4}{5}}$

2. Guess a formula (in terms of n) for the sum

$$S_n = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}.$$

Examples above suggest this is $\boxed{\frac{n}{n+1}}$

3. Using your formula, find a formula for $S_n + \frac{1}{(n+1)(n+2)}$. What do you observe?

$$\frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)}$$

$$= \frac{n+1}{n+2}.$$

Observe: this is the same formula, but with n replaced by $n+1$.