1. [12 points] A five-way prime number race is held, as follows. There are five teams, called Team 0, Team 1, Team 2, Team 3, and Team 4. The game consists of a sequence of rounds, numbered $1, 2, 3 \cdots$. In round n,

   - If n is prime and $n \equiv a \pmod 5$ (for $0 \le a < 5$) then Team a scores a point.
   - If n is not prime, no one scores a point.

   Determine which team is ahead after 50 rounds.

2. [12 points] Let a, b, m, n be integers such that

$$a \equiv b \pmod{mn}.$$

   Prove that also

$$a \equiv b \pmod{m}.$$

3. (a) [8 points] Find integers u, v such that $80u + 523v = 1$.

   (b) [4 points] Find a second pair of integers u', v' solving $80u' + 523v' = 1$, but in which u has the opposite sign. That is, if you found $u > 0$ in part (a), find a solution with $u' < 0$ in this part, and vice versa.

4. [12 points] Solve the linear congruence

$$12x \equiv 6 \pmod{105}.$$

   Your answer should describe all solutions to the congruence, and may be stated as a congruence $x \equiv \ldots \pmod{\ldots}$.

5. (a) [4 points] Prove that if n is any integer, then $n^2 \not\equiv 2 \pmod 3$.

   (b) [8 points] Prove that if (a, b, c) is a Pythagorean triple (a positive integer solution to $a^2 + b^2 = c^2$), then $3 \mid ab$.