

1. [12 points] Find all prime numbers p between 1 and 100 such that

$$p \equiv -1 \pmod{15}.$$

2. [12 points] Recall that a *primitive Pythagorean triple* consists of three positive integers (a, b, c) such that

- $a^2 + b^2 = c^2$, and
- there are no common factors of a, b and c .

Find a primitive Pythagorean triple such that $a = 15$.

3. [12 points] Compute the greatest common divisor of 1106 and 203.
4. [12 points] Solve the following congruence.

$$28x \equiv 3 \pmod{149}$$

5. [12 points] Suppose that a, b, c are positive integers such that $\gcd(a, b) = 1$. Prove that if a divides bc , then a divides c .
6. [12 points] Suppose that you enter a store carrying a large supply of 6 dollar coins. The shopkeeper is able to make change using 28 dollar coins and 63 dollar coins. Find a way that you can purchase a 1 dollar item.

For partial credit, you may first assume that both you and the shopkeeper have a large supply of all three types of coins (6, 28, and 63) and solve the problem in this context.