Amherst College
Department of Mathematics and Statistics

MATH 250 MIDTERM 1 PRACTICE 28 FEBRUARY 2025

NAME:_____

Read This First!

- The exam uses **both sides of the page**.
- Keep cell phones off and out of sight.
- Do not talk during the exam.
- You are allowed one page of notes, front and back. No other books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Show **ALL** work clearly in the space provided or on the blank pages.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- You may cite any theorems proved in class or on the homework in your proofs, except in cases where the statement to be proved is essentially the same as a theorem proved earlier. In that case you should write out the full proof. Please ask me if you are uncertain about whether you should prove a theorem or if it is enough to cite it.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	6	$\sum$
Points:	12	12	12	12	12	12	72
Score:							

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1. [12 points] Find all prime numbers p between 1 and 100 such that

$$p \equiv -1 \pmod{15}.$$

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2. [12 points] Recall that a *primitive Pythagorean triple* consists of three positive integers (a, b, c) such that

   - $a^2 + b^2 = c^2$, and
   - there are no common factors of a, b and c.

   Find a primitive Pythagorean triple such that $a = 15$.

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3. [12 points] Compute the greatest common divisor of 1106 and 203.

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4. [12 points] Solve the following congruence.

$$28x \equiv 3 \pmod{149}$$

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5. [12 points] Suppose that a, b, c are positive integers such that $\gcd(a, b) = 1$. Prove that if a divides bc, then a divides c.

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6. [12 points] Suppose that you enter a store carrying a large supply of 6 dollar coins. The shopkeeper is able to make change using 28 dollar coins and 63 dollar coins. Find a way that you can purchase a 1 dollar item.

   For partial credit, you may first assume that both you and the shopkeeper have a large supply of all three types of coins (6,28, and 63) and solve the problem in this context.

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