Think of each problem set as a study aid, more so than just an assignment. To help with this, I'll list the main points that you should try to master in a given week at the top of the problem set, as "study guide," together with the relevant textbook section.

Study guide

- (§1) The main theme of §1 is that number theory is partly experimental, and partly theoretical. Look over the five steps on pp. 10-11 and practice them in the examples in the book and here on the problem set.
- (§2) The definition of *primitive Pythagorean triple* (p. 15) and the reasons for each part of it.
- (§2) How can you *use* the classification of primitive Pythagorean triples in Theorem 2.1 to find Pythagorean triples with certain properties?
- (§2) Practice explaining the proof of Theorem 2.1 (classification of Pythagorean triples).
- (§3) How can you use the approach of §3 to find all *rational* solutions to $x^2 + y^2 = 1$ and similar equations?
- (§3) Why do *rational* solutions to $x^2 + y^2 = 1$ give another way to find (integer!) Pythagorean triples?

1. Let $S(n)$ denote the sum of the first n odd numbers (for example, $S(3) = 1 + 3 + 5 = 9$). Compute $S(1), S(2), S(3), S(4)$, and $S(5)$. What pattern do these numbers follow?
2. Find two pairs (a, b) of positive (in particular, nonzero) whole numbers satisfying the equation
$$a^2 - 2b^2 = 1.$$
3. A *prime number race* is held, as follows. There are two teams, called Team 1 and Team 2. The game consists of a sequence of rounds, numbered $1, 2, 3$, and so on. In round n:
   - If n is prime and leaves a remainder of 1 when divided by 3, Team 1 scores a point.
   - If n is prime and leaves a remainder of 2 when divided by 3, Team 2 scores a point.
   - If n is not prime, or n is divisible by 3, neither team scores a point (recall that 1 is not considered prime).

   Determine the score of the game after 120 rounds.

   Hint Write down a list of all the primes up to 120, e.g. using the Sieve of Eratosthenes. Use this list to determine the points scored by each team.
4. Prove that the numbers $\sqrt{3}$ and $\sqrt[3]{2}$ are both irrational.
5. Recall that a *primitive Pythagorean triple* consists of three positive integers (a, b, c) such that $a^2 + b^2 = c^2$, and there is no common factor of a, b, and c.
   (a) Find a primitive Pythagorean triple with $a = 33$.
   (b) Find a primitive Pythagorean triple with $c = 85$.
   (c) Find another possible answer to either part (a) or part (b).

Note. In part (c), you answer should not simply swap the numbers a and b (for example, if $(3,4,5)$ were an answer to (a), then do not give $(4,3,5)$ for (c)).

6. (a) Follow the method of chapter 3 to describe all the *rational* solutions to the equation $x^2+y^2=2$. Note that you will need to choose a different "center of projection" than was used for the equation $x^2+y^2=1$.

   (b) Using your answer to part (a), find an *natural number* (i.e. positive integer) solution to the equation $a^2+b^2=2c^2$, such that $a \neq b$.