Study guide

- (§14) Know the definition of a Mersenne prime.
- (§14) Be able to prove: if $2^p - 1$ is prime, then p is prime as well.
- (§15) What is the link between Mersenne primes and (even) perfect numbers?
- (§15) Understand the importance of *multiplicativity* in the study of perfect numbers.
- (§16) How can you use the *successive squaring* technique to quickly compute modular powers (by hand if needed)?

1. (Textbook 14.1)
   Prove that if $a \geq 2$ and $n \geq 1$ are integers, and $a^n + 1$ is prime, then n is a power of 2.

2. (Textbook 14.2, on Fermat primes)

3. (Textbook 14.3, on primes of the form $\frac{1}{2}(3^n - 1)$)

4. (Textbook 15.3, on the impossibility of certain odd perfect numbers)

5. (Textbook 15.8, on "amicable numbers")

6. Evaluate the following using the method of successive squaring. Try to do as much by hand as possible, but use a calcualtor as needed (look for ways to make your calculations easier where you can). Note that on exams, I will often provide a multiplication table for modular arithmetic if the modulus is larger than 20 or so, so you won't have to do anything quite this complicated by hand under time pressure.

   (a) The remainder when 2^{25} is divided by 29.

   (b) The last two digits of 29^{72}.

   (c) The remainder when 49^{37} is divided by 101.