

(This is a modified version of Harris Daniels's Midterm 1 practice test from Fall 2018)

1. TRUE OR FALSE: (No justification necessary.)

(a) [3 points] There exists a set of three vectors $\{\vec{u}, \vec{v}, \vec{w}\} \subseteq \mathbb{R}^2$ that is linearly independent. **T** **F**

(b) [3 points] Every system of linear equations has at least one solution. **T** **F**

(c) [3 points] The set of solutions to a system of linear equations in n unknowns is a subspace of \mathbb{R}^n . **T** **F**

(d) [3 points] Denote by $C^1(\mathbb{R})$ the subspace of $F(\mathbb{R})$ consisting of differentiable functions. The set $\{f \in C^1(\mathbb{R}) \mid f + f' = 0\}$ is a subspace of $C^1(\mathbb{R})$. **T** **F**

2. Let V be a vector space, and let $S \subseteq V$. Define the following terms and phrases. You may use other standard terms without defining them.

(a) [5 points] The set S spans V .

(b) [5 points] The set S is linearly independent.

(c) [5 points] The set S is a basis of V .

3. [15 points] Find a set of vectors spanning the set of solutions to the following system of equations.

$$\begin{cases} x_1 + 2x_2 & - x_4 = 0 \\ -2x_1 - 3x_2 + 4x_3 - 5x_4 = 0 \\ 2x_1 + 4x_2 & - 2x_4 = 0 \end{cases}$$

4. (a) [5 points] Let $S_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\} \subseteq \mathbb{R}^3$. Is S_1 linearly independent? Why or why not?

- (b) [5 points] Let $S_2 = \{1 + x + x^2, 2 - x, 3 - 2x + x^2, x - 2x^2\} \subseteq P_2(\mathbb{R})$. Is S_2 linearly independent? Why or why not?

5. [15 points] Let $S = \{1 + 2x + 3x^2, 1 + x^2 + x^3, x^2 + x^3\} \subseteq P_3(\mathbb{R})$. Is $3 + 2x + 4x^2 + x^3 \in \text{Span}(S)$? Justify your answer.

6. [15 points] Let V be a vector space and let S and T be subsets of V . Show that if $\text{Span}(S) = V$ and $S \subseteq \text{Span}(T)$, then $\text{Span}(T) = V$.