Amherst College
Department of Mathematics

Math 271 — Midterm 2 Practice Exam 1 — Spring 2022

Name:____

This is a modified version of a practice exam from Fall 2016.

Read This First!

- Please read each question carefully. Show **ALL** work clearly in the space provided.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question.
- Please cross out or fully erase any work that you do not want graded.
- The point value of each question is indicated after its statement.
- No books or other references are permitted.
- Calculators are not allowed and you must show all your work.

Grading - For Administrative Use Only

Question:	1	2	3	4	5	Total
Points:	20	15	0	10	10	55
Score:						

1. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear, $T((1,0)) = (1,2)$, and $T((1,1)) = (3,5)$.
   (a) What is $T((2,3))$? [5]
   (b) Is T injective? [5]
   (c) Let α and β be the standard basis for $\mathbb{R}^2$. Compute $[T]_\alpha^\beta$. [10]

2. Let $V = P_2(\mathbb{R})$, $W = \mathbb{R}^2$, $\alpha = \{1, 1+x, 1+x+x^2\}$, and β is the standard basis for $\mathbb{R}^2$. Suppose that $T : V \to W$ is a linear transformation such that $[T]_\alpha^\beta = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$.
   (a) Find a basis for $\mathrm{Ker}(T)$. [10]
   (b) Is T surjective? Justify your answer. [5]

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $T((a_1, a_2, a_3)) = (a_1 + a_2, a_2, a_1 - a_3)$. Show that T is a invertible.

4. Let $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = 0\}$. Prove that V is isomorphic to $\mathbb{R}^2$. [10]

5. Suppose that $T : V \to W$ is a linear transformation such that $\dim \ker T = 0$. Prove that if $\vec{v}_1, \vec{v}_2 \in V$ satisfy $T(\vec{v}_1) = T(\vec{v}_2)$, then $\vec{v}_1 = \vec{v}_2$. [10]