Amherst College
Department of Mathematics and Statistics

MATH 271-02 — MIDTERM 3 PRACTICE — SPRING 2022

NAME:____

Read This First!

- Keep cell phones off and out of sight.
- Do not talk during the exam.
- You are allowed one page of notes, front and back. No other books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Please read each question carefully. Show **ALL** work clearly in the space provided or on the blank pages.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- You may cite any theorems proved in class or on the homework in your proofs, except in cases where the statement to be proved is essentially the same as a theorem proved earlier. In that case you should write out the full proof. Please ask me if you are uncertain about whether you should prove a theorem or if it is enough to cite it.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	6	Total
Points:	15	15	15	15	15	10	85
Score:							

1. [15 points] Let $V = \mathbb{R}^3$, $\alpha = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$, and $\beta = \{(2, 1, 0), (-2, 0, 1), (-1, 0, 1)\}$. Find the change of basis matrix from α to β.

2. [15 points] Let $A = \begin{bmatrix} 3 & 8 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ -25 & -66 & -1 & -3 \\ -9 & -24 & 0 & -1 \end{bmatrix}$. Compute $\det(A)$.

3. Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix}$.

(a) [5 points] Find the eigenvalues of A.

(b) [5 points] Find a basis for each eigenspace of A.

(c) [5 points] Is A diagonalizable? Why or why not?

4. [15 points] Prove that if A is an $n \times n$ diagonalizable matrix, then A^3 is also diagonalizable.

5. [15 points] Prove that if $\vec{u}, \vec{v}$ are any two vectors in $\mathbb{R}^n$, then

$$\|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2 = 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2.$$

Here vertical bars $\|\cdots\|$ indicate the norm (length) of a vector in $\mathbb{R}^n$.

6. [10 points] Suppose that $A, B \in M_{n\times n}(\mathbb{R})$. Prove that if $\det(AB) = 0$, then either A or B is not invertible.