Note. This "problem set" need not be handed in, since you have an exam this week. You should still work through the problems, but rather than requiring you to submit solutions, I will post sample solutions in the middle of next week for you to check over your work.

Textbook reading for this week:

- §4.3 (geometry in $\mathbb{R}^n$)
- (Not required for the last exam) the last several classes will discuss material up to the *spectral theorem for symmetric matrices*, which provides a theoretical foundation for principal components analysis (as described in the guest lecture earlier). In our book, this material occupies sections §4.4-4.6, so you may be interested in skimming those sections to learn more.

Study items:

- Decide if a given matrix, or linear transformation, is diagonalizable.
- For a diagonalizable matrix A, find an invertible matrix Q and diagonal matrix D such that $A = QDQ^{-1}$.
- Calculate the geometric and algebraic multiplicity of an eigenvalue of a matrix or linear transformation.
- Know the definition of the *standard inner product* or *dot product* on $\mathbb{R}^n$, and the two most common notations for it: $\langle \vec{v}, \vec{w} \rangle$ and $\vec{v}\dot{\vec{w}}$.
- How do you use the dot product to measure lengths and angles in $\mathbb{R}^n$?
- What does it mean for a set of vectors to be *orthogonal* or *orthonormal*? Why does this imply that they are linearly independent?

Problems:

1. *(Damiano–Little 4.2.1(a))* (determine whether 2×2 matrix is diagonalizable, and find basis of eigenvalues if so)
2. *(Damiano–Little 4.2.3)* (necessary and sufficient conditions for a 3×3 upper-triangular matrix to be diagonalizable)
3. *(Damiano–Little 4.2.6(a))* (if A is diagonalizable, then so is A^k)
4. *(Damiano–Little 4.2.14(a))* (eigenvectors and commuting matrices)
5. Calculate the matrix A^{10} where A is the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$. (Hint: use the fact that A is diagonalizable to write A as QDQ^{-1}. Also: $2^{10} = 1024$.)
6. *(Damiano–Little 4.3.1(a,b,c))* (inner products and norms in $\mathbb{R}^3$)
7. *(Damiano–Little 4.3.7)* (determine whether a set of vectors is orthogonal)
8. *(Damiano–Little 4.3.8)* (an orthogonal set of vectors is linearly independent)

9. *(Damiano–Little 4.3.4)* **This originally had the wrong problem number – 4.4.4 instead of 4.3.4. I'm sorry about this error!** (if vectors are orthogonal, then a Pythagorean theorem holds)

10. *(Damiano–Little Ch 4 supplemental 4(a,c))* (diagonalizable? If so, give diagonal matrix and change of basis; 2×2 and 3×3)