Note. This “problem set” need not be handed in, since you have an exam this week. You should still work through the problems, but rather than requiring you to submit solutions, I will post sample solutions in the middle of next week for you to check over your work.

Textbook reading for this week:

- §4.3 (geometry in $\mathbb{R}^n$)
- (Not required for the last exam) the last several classes will discuss material up to the spectral theorem for symmetric matrices, which provides a theoretical foundation for principal components analysis (as described in the guest lecture earlier). In our book, this material occupies sections §4.4-4.6, so you may be interested in skimming those sections to learn more.

Study items:

- Decide if a given matrix, or linear transformation, is diagonalizable.
- For a diagonalizable matrix $A$, find an invertible matrix $Q$ and diagonal matrix $D$ such that $A = QDQ^{-1}$.
- Calculate the geometric and algebraic multiplicity of an eigenvalue of a matrix or linear transformation.
- Know the definition of the standard inner product or dot product on $\mathbb{R}^n$, and the two most common notations for it: $\langle \vec{v}, \vec{w} \rangle$ and $\vec{v} \cdot \vec{w}$.
- How do you use the dot product to measure lengths and angles in $\mathbb{R}^n$?
- What does it mean for a set of vectors to be orthogonal or orthonormal? Why does this imply that they are linearly independent?

Problems:

1. *(Damiano–Little 4.2.1(a)) (determine whether 2 × 2 matrix is diagonalizable, and find basis of eigenvalues if so)*

2. *(Damiano–Little 4.2.3) (necessary and sufficient conditions for a 3 × 3 upper-triangular matrix to be diagonalizable)*

3. *(Damiano–Little 4.2.6(a)) (if $A$ is diagonalizable, then so is $A^k$)*

4. *(Damiano–Little 4.2.14(a)) (eigenvectors and commuting matrices)*

5. Calculate the matrix $A^{10}$ where $A$ is the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$. (Hint: use the fact that $A$ is diagonalizable to write $A$ as $QDQ^{-1}$. Also: $2^{10} = 1024$.)

6. *(Damiano–Little 4.3.1(a,b,c)) (inner products and norms in $\mathbb{R}^3$)*

7. *(Damiano–Little 4.3.7) (determine whether a set of vectors is orthogonal)*

8. *(Damiano–Little 4.3.8) (an orthogonal set of vectors is linearly independent)*
9. *(Damiano–Little 4.3.4)* This originally had the wrong problem number — 4.4.4 instead of 4.3.4. I’m sorry about this error! (If vectors are orthogonal, then a Pythagorean theorem holds)

10. *(Damiano–Little Ch 4 supplemental 4(a,c))* (Diagonalizable? If so, give diagonal matrix and change of basis; $2 \times 2$ and $3 \times 3$)