Textbook reading for this week:

- Chapter 1.3 (Linear combinations) and 1.4 (linear independence)
- Sections §A1.1 through §A1.4 (pages 344-351) of Appendix 1 (notation about sets and logic)

Study items:

- Definitions: linear combination, span.
- Understand what is means to say that a subspace is spanned by a set of vectors.
- Definitions: linearly dependent, linearly independent.
- How do you determine whether a set of vectors is linearly dependent or linearly independent?

Problems:

1. (Damiano–Little 1.2.3(h,i)) For each of the following subsets $W$ of a vector space $V$, determine if $W$ is a subspace of $V$. Say why or why not in each case.
   - (h) $V = P_n(\mathbb{R})$, $W = \{p : p(\sqrt{2}) = 0\}$.
   - (i) $V = P_n(\mathbb{R})$, $W = \{p : p(1) = 1$ and $p(2) = 0\}$.

2. (Damiano–Little 1.2.5) Let $W$ be a subspace of a vector space $V$, let $\vec{y} \in V$, and define the set $\vec{y} + W = \{\vec{x} \in V : \vec{x} = \vec{y} + \vec{w}$ for some $\vec{w} \in W\}$. Show that $\vec{y} + W$ is a subspace of $V$ if and only if $\vec{y} \in W$.

3. (Damiano–Little 1.3.1(a,d))
   - (a) Let $S = \{(1,0,0), (0,1,2)\}$ in $\mathbb{R}^3$. Describe the set $\text{Span}(S)$. (See the examples in this section to see what is meant by “describe” in questions like this)
   - (d) Let $S = \{1, x, x^2\}$. Describe the set $\text{Span}(S)$ in $P_2(\mathbb{R})$.

4. (Damiano–Little 1.3.3) In $V = P_2(\mathbb{R})$, let $S = \{1, 1 + x, 1 + x + x^2\}$. Show that $\text{Span}(S) = P_2(\mathbb{R})$.

5. (Damiano–Little 1.3.6(a)) Let $W_1 = \text{Span}(S_1)$ and $W_2 = \text{Span}(S_2)$ be subspaces of a vector space. Show that $W_1 \cap W_2 \supseteq \text{Span}(S_1 \cap S_2)$.

Note. This problem uses the symbol $\cap$ for the “intersection” of two sets. See p. 350 of the textbook (in Appendix 1) for a discussion of this notation and several related symbols that we may use throughout the course.

6. (Damiano–Little 1.3.7) Show that if $S$ is a subset of a vector space $V$ and $W$ is a subspace of $V$ that contains $S$, then $\text{Span}(S) \subseteq W$.

7. (Damiano–Little 1.4.1(a,b,c,e)) Determine whether each of the following sets of vectors is linearly dependent or linearly independent.
   - (a) $S = \{(1,1), (1,3), (0,2)\}$ in $\mathbb{R}^2$
   - (b) $S = \{(1,2,1), (1,3,0)\}$ in $\mathbb{R}^3$
(c) \( S = \{(0, 0, 0), (1, 1, 1)\} \) in \( \mathbb{R}^3 \)

(e) \( S = \{(x^2 + 1, x - 7)\} \) in \( P_2(\mathbb{R}) \)

8. (Damiano–Little 1.4.5) Let \( \vec{v}, \vec{w} \in V \) Show that \( \{\vec{v}, \vec{w}\} \) is linearly independent if and only if \( \{\vec{v} + \vec{w}, \vec{v} - \vec{w}\} \) is linearly independent.

9. Suppose that \( \{\vec{u}, \vec{v}, \vec{w}\} \) is a set of three vectors in a vector space \( V \), and that \( \vec{x} \in \text{Span} (\{\vec{u}, \vec{v}, \vec{w}\}) \).
Prove that \( \{\vec{u}, \vec{v}, \vec{w}, \vec{x}\} \) is a linearly dependent set.

**Extra practice** (not to hand in)

- (Damiano–Little 1.2.6)
- (Damiano–Little 1.2.10)
- (Damiano–Little 1.3.6)
- (Damiano–Little 1.3.8)
- (Damiano–Little 1.4.2)