Textbook reading for this week:

- Chapter 1.3 (Linear combinations) and 1.4 (linear independence)
- Sections §A1.1 through §A1.4 (pages 344-351) of Appendix 1 (notation about sets and logic)

Study items:

- Definitions: *linear combination*, *span*.
- Understand what is means to say that a subspace is spanned by a set of vectors.
- Definitions: *linearly dependent*, *linearly independent*.
- How do you determine whether a set of vectors is linearly dependent or linearly independent?

Problems:

1. *(Damiano–Little 1.2.3(h,i))* For each of the following subsets W of a vector space V, determine if W is a subspace of V. Say why or why not in each case.

   (h) $V = P_n(\mathbb{R})$, $W = \{p : p(\sqrt{2}) = 0\}$.

   (i) $V = P_n(\mathbb{R})$, $W = \{p : p(1) = 1 \text{ and } p(2) = 0\}$.

2. *(Damiano–Little 1.2.5)* Let W be a subspace of a vector space V, let $\vec{y} \in V$, and define the set $\vec{y} + W = \{\vec{x} \in V : \vec{x} = \vec{y} + \vec{w} \text{ for some } \vec{w} \in W\}$. Show that $\vec{y} + W$ is a subspace of V if and only if $\vec{y} \in W$.

3. *(Damiano–Little 1.3.1(a,d))*

   (a) Let $S = \{(1, 0, 0), (0, 0, 2)\}$ in $\mathbb{R}^3$. Describe the set $\mathrm{Span}(S)$. (See the examples in this section to see what is meant by "describe" in questions like this)

   (d) Let $S = \{1, x, x^2\}$. Describe the set $\mathrm{Span}(S)$ in $P_4(\mathbb{R})$.

4. *(Damiano–Little 1.3.3)* In $V = P_2(\mathbb{R})$, let $S = \{1, 1 + x, 1 + x + x^2\}$. Show that $\mathrm{Span}(S) = P_2(\mathbb{R})$.

5. *(Damiano–Little 1.3.6(a))* Let $W_1 = \mathrm{Span}(S_1)$ and $W_2 = \mathrm{Span}(S_2)$ be subspaces of a vector space. Show that $W_1 \cap W_2 \supseteq \mathrm{Span}(S_1 \cap S_2)$.

Note. This problem uses the symbol $\cap$ for the "intersection" of two sets. See p. 350 of the textbook (in Appendix 1) for a discussion of this notation and several related symbols that we may use throughout the course.

6. *(Damiano–Little 1.3.7)* Show that if S is a subset of a vector space V and W is a subspace of V that contains S, then $\mathrm{Span}(S) \subset W$.

7. *(Damiano–Little 1.4.1(a,b,c,e))* Determine whether each of the following sets of vectors is linearly dependent or linearly independent.

   (a) $S = \{(1, 1), (1, 3), (0, 2)\}$ in $\mathbb{R}^2$

   (b) $S = \{(1, 2, 1), (1, 3, 0)\}$ in $\mathbb{R}^3$

(c) $S = \{(0,0,0),(1,1,1)\}$ in $\mathbb{R}^3$

(e) $S = \{(x^2+1, x-7\}$ in $P_2(\mathbb{R})$

8. *(Damiano–Little 1.4.5)* Let $\vec{v}, \vec{w} \in V$ Show that $\{\vec{v}, \vec{w}\}$ is linearly independent if and only if $\{\vec{v}+\vec{w}, \vec{v}-\vec{w}\}$ is linearly independent.

9. Suppose that $\{\vec{u}, \vec{v}, \vec{w}\}$ is a set of three vectors in a vector space V, and that $\vec{x} \in \operatorname{Span}(\{\vec{u}, \vec{v}, \vec{w}\})$. Prove that $\{\vec{u}, \vec{v}, \vec{w}, \vec{x}\}$ is a linearly dependent set.

Extra practice (not to hand in)

- *(Damiano–Little 1.2.6)*
- *(Damiano–Little 1.2.10)*
- *(Damiano–Little 1.3.6)*
- *(Damiano–Little 1.3.8)*
- *(Damiano–Little 1.4.2)*