Textbook reading for this week:

- Chapter 1.5 (solving systems of linear equations).
- Suggested: begin reading Chapter 1.6 (basis and dimension). There are no problems about that section on this assignment, but we will begin it in class.

Study items:

- Terminology: linear systems of equations, augmented matrix, echelon form, free variables, bound variables, general solution.
- Procedure: using the *elimination method* to simplify a given system of linear equations to echelon form.
- Procedure: find the general solution to a system of linear equations (this is also called "parameterizing the solutions," which is how the book often phrases it).
- How can you use a system of linear equations to (a) determine whether a vector is in the span of a given set of vectors, and (b) determine whether a given set of vectors is linearly independent?
- How do you tell that a linear system has no solutions? Has a unique solution? Has infinitely many solutions?

Problems:

1. Linear independence has a simple description for sets of size 1 or 2.
   (a) Let $\vec{v}$ be a vector in a vector space V. Prove that $\{\vec{v}\}$ is linearly independent if and only if $\vec{v} \neq \vec{0}$.
   (b) Let $\vec{v}, \vec{w}$ be two vectors in a vector space V. Prove that $\{\vec{v}, \vec{w}\}$ is linearly dependent if and only if one is a scalar multiple of the other (that is, either there exists $c \in \mathbb{R}$ such that $\vec{w} = c\vec{v}$, or there exists $c \in \mathbb{R}$ such that $\vec{v} = c\vec{w}$).
   (c) Give an example of two vectors $\vec{v}, \vec{w}$ in $\mathbb{R}^2$ such that $\vec{v}$ is *not* a scalar multiple of $\vec{w}$, but $\{\vec{v}, \vec{w}\}$ is linearly dependent. Explain why this does not contradict part (b).
2. *(Damiano–Little 1.4.1(d,f,i,j))* (Determine whether sets of vectors or linearly dependent or linearly independent)
3. *(Damiano–Little 1.4.8)* Let W_1, W_2 be subspaces of a vector space, satisfying $W_1 \cap W_2 = \{\vec{0}\}$. Show that if $S_1 \subset W_1$ and $S_2 \subset W_2$ are linearly independent sets, then their union $S_1 \cup S_2$ is linearly independent.
4. Suppose that $\vec{v}_1, \cdots, \vec{v}_n$ are vectors in a vector space V, and assume that $\{\vec{v}_1, \cdots, \vec{v}_{n-1}\}$ is a linearly independent set. Prove that $\{\vec{v}_1, \cdots, \vec{v}_n\}$ is also a linearly independent set if and only if $\vec{v}_n \notin \text{Span}\{\vec{v}_1, \cdots, \vec{v}_{n-1}\}$.
5. Let S be a nonempty linearly independent subset of a vector space V . Prove that every vector in Span(S) can be written as a *unique* linear combination of vectors in S.

Note. (3/3/22) You may assume that S is a *finite* set, and begin by writing $S = \{\vec{v}_1, \cdots, \vec{v}_n\}$ if you wish, since we have not placed much emphasis on spans of infinite sets.

6. *(Damiano–Little 1.5.1(b,d,e))* (Find the general solution of several linear systems)

Note. When the textbook says "find a parameterization of the set of solutions," this means the same thing as "find the general solution," which is the wording we used in class on Wednesday 2/23.

7. *(Damiano–Little 1.5.2(a,b))* (Find the set of vectors spanning the subspace determined by the given set of linear equations)

8. *(Damiano–Little 1.5.3(a,b))* (Determine whether a given vector $\vec{v}$ is in the span of a given set S in $\mathbb{R}^3$)

9. *(Damiano–Little 1.5.5(a,c))* (Determine whether or not a given set of vectors is linearly independent)

Extra practice (not to hand in)

- *(Damiano–Little 1.4.8)*
- *(Damiano–Little 1.4.10)*
- *(Damiano–Little 1.4.11)*