Textbook reading for this week:

- §2.1 (linear transformations)
- §2.2 (matrix representations)

Study items:

- How do you show that a given function either is or is not a linear transformation?
- A linear transformation is determined by its values on the objects of a basis. Why?
- How can you calculate the matrix associated to a linear transformation with respect to given ordered bases?
- Notation: writing vectors in $\mathbb{R}^n$ as column vectors.
- Know the definition of matrix multiplication.
- How does a $m \times n$ matrix encode a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$?

Problems:

1. Prove the following "2 out of 3 theorem": if S is a finite subset of a vector space, and *any two* of the following three statements is true, then S is a basis of V.
   - S is linearly independent.
   - S spans V.
   - S has exactly $\dim V$ elements.

   (This theorem amounts to three different statements that you should prove separately. One statement follows immediately from the definition of "basis.")

Note. This "2 out of 3 theorem" is a useful shortcut in checking that a given set is a basis of a vector space, since it can be tedious to check both linearly independence and span, but it can be much easier to just count elements. The next problem is an example.

2. (a) Prove that $S = \{(2,1),(3,2)\}$ is a linearly independent set in $\mathbb{R}^2$.
   (b) Prove using the "2 out of 3 theorem" above that S is a basis of $\mathbb{R}^2$. (This should only require one sentence, and no computations)
3. *(Damiano–Little 2.1.3(a,b,e,f,g))* (Determine whether a given function is a linear transformation)
4. *(Damiano–Little 2.1.4)* (Describing linear transformations $\mathbb{R}^n \to \mathbb{R}$)
5. *(Damiano–Little 2.1.8)* (Some properties of rotations R_θ)
6. *(Damiano–Little 2.1.11)* (Infer $T(2,2)$ from $T(1,1)$ and $T(3,0)$)
7. Suppose that $T : V \to W$ is a linear transformation, $S = \{\vec{v}_1, \cdots, \vec{v}_n\}$ is a set of vectors in V, and $T = \{T(\vec{v}_1), \cdots, T(\vec{v}_n)\}$.

(a) Prove that if T is linearly independent, then S is linearly independent.

(b) Show that the converse statement is false (write down a specific linear transformation and sets S and T where S is linearly independent but T is not).

8. *(Damiano–Little 2.2.3)* (Matrix representations of some linear transformations)
9. *(Damiano–Little 2.2.7)* (Some matrix mulitplication computations)
10. *(Damiano–Little 2.2.8(a))* (Rotation via matrix multiplication)

Extra practice (not to hand in)

- *(Damiano–Little 2.1.3(remaining parts))*
- *(Damiano–Little 2.1.5(remaining parts))*
- *(Damiano–Little 2.1.10)*
- *(Damiano–Little 2.2.12(b,c))*
- *(Damiano–Little 2.2.13)*